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Event-triggered adaptive consensus for stochastic multi-agent systems with saturated input and partial state constraints

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ABSTRACT

This paper is concerned with the consensus tracking problem of stochastic multi-agent systems with both output, partial state constraints, and input saturation via event-triggered strategy. To handle with the saturated control inputs, the saturation function is transformed into a linear form of the control input. By using radial basis function neural network to approximate the unknown nonlinear function, the unmeasurable states are acquired by an adaptive observer. To ensure that the constraints of system outputs and partial states are never violated, an appropriate time-varying barrier Lyapunov function is constructed. The control scheme is event-triggered in order to save communication resources. The proposed distributed controller can guarantee the boundedness of all system signals, the consensus tracking with a small bounded error, and the avoidance of the Zeno behavior by using backstepping techniques. The validity of the theoretical results is verified by computer simulation.

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1. Introduction

A multi-agent system (MAS) consists of a group of agents in which the communication relationship among agents is modeled by a directed or undirected graph. The research of multi-agent systems (MASs) has attracted extensive attention due to the autonomy, fault tolerance, flexibility and cooperation of MASs in complex tasks. Therefore, MASs have been widely used in different fields such as distributed optimization [1,2], unmanned air vehicles [3], tracking control [4,5] and so on. Consensus problem is a key problem in MASs. By distributed coordination, a MAS can for example reach the same state, the same speed, the same position and the same attitude. So far, the consensus problem of MASs has been studied by many scholars, such as in [6–11] and so on.

Due to uncertainty in reality, stochastic system models have been widely used in many fields, such as chemistry, finance, physics and neuroscience [12–14] and so on. Therefore, the research of stochastic multi-agent systems (SMASs) has become a hot topic in control community. Compared with deterministic MASs, it is more challenging to implement consensus tracking in SMASs. In spite of this, some scholars have made considerable progress in this filed. The consensus problem of SMASs with delay and noise was studied in [15]. The distributed output synchronization of a class of nonlinear high-order SMASs

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with directed network topology was studied in [16,17]. In [18], the authors proposed an impulsive consensus protocol for perturbed nonlinear SMASs using comparison system method.

In MASs, an important problem is how to reduce the consumption of limited network resources. The practice proves that the event-triggered control (ETC) method is a good choice. The ETC schemes have been proposed as an alternative to the classical periodic control schemes. The control signal is updated only when the designed triggering rules are violated. Due to its advantages, the ETC schemes have been widely used in SMASs [19–26]. The consensus problem of nonlinear non-affine pure-feedback SMASs was studied in [19] and a fuzzy adaptive quantized ETC scheme was proposed. The event-triggered tracking control problem of a class of high order nonlinear SMASs was studied in [20]. In [21], the authors studied the consensus tracking problem of continuously switched SMASs with an ETC strategy. In [22], the authors studied the consensus problem for discrete time SMASs with noises via ETC strategy. In [23], the authors studied the leader-following consensus problem for a class of high order SMASs via output feedback, both event-triggered and self-triggered control schemes were proposed in undirected networks. A distributed ETC strategy was used in [24] to study the mean square consensus problem of SMASs. A new centralized/distributed hybrid ETC strategy was proposed in [25] for leader-following SMASs. The consensus problem of time-varying discrete-time SMASs with sensor saturation was studied in [26] using an ETC scheme. The consensus tracking problem for a class of continuous switching nonlinear SMASs with an ETC stat-egy was investigated in [27]. The adaptive bipartite containment control problem for nonlinear SMASs with an event-triggered mechanism was investigated in [28].

To ensure system efficiency and security, it is necessary to constrain the state and output of the system. To solve the constraint problem, some scholars have conducted a lot of research and put forward some effective methods [29,30]. In these studies, the barrier Lyapunov function (BLF) method has become a common method for state or output constrained systems, and the stabilization problem for a class of feedback systems with multi-state constraints was proposed for the first time in [31]. Since then, the BLF method has been widely used in systems with various constraints, such as constant constraints [32,33] and time-varying constraints [34–37]. With asymmetric input dead zones, output constraints and system uncertainties, the problem of adaptive neural network control for vibrating flexible string systems was investigated in [32]. A new control algorithm was proposed in [35] for a class of SMASs with time-varying output tracking constraints. By an adaptive ETC strategy, the control problem of nonlinear systems with time-varying partial state constraints (PSCs) and input saturation was studied in [36]. An adaptive neural network control scheme was proposed in [37] for a class of SMASs with time-varying full-state constraints (FSCs). Based on the above literature analysis, there are few researches on SMASs that consider PSCs using ETC strategies which motivates the research of this paper.

In this paper, a consensus tracking control scheme for SMASs with unknown nonlinearity and external disturbance is proposed in directed networks containing a spanning tree. The output and partial states of the SMASs are constrained by pre-specified boundary functions. A distributed control scheme is developed using an ETC strategy. A state observer is designed to deal with the unmeasurable states of the system. Moreover, in some actual physical systems, the problem of input saturation [38,39] is often encountered. When the input is saturated, the performance of the system will become poor or even unstable. The input saturation problem is also considered in this paper and the designed saturated controller can ensure the achievements of the control objectives. Compared with the existing works, the contributions of this paper are mainly reflected in the following points:

(i) An adaptive distributed ETC scheme with observer is proposed for SMASs with PSCs and input saturation, which can guarantee the consensus tracking with a small bounded error, the boundedness of all system signals and the avoidance of the Zeno behavior. Compared with the time-triggered algorithms for SMASs [16,17,37], the event-triggered schemes [19–26] can reduce the communication burden.

(ii) Different from the control schemes [16,18] for SMASs, the unknown nonlinearity and the external disturbance are both taken into account in this paper via output feedback in directed networks including a spanning tree. So the model considered is more general. The unknown nonlinear dynamics are approximated by radial basis function neural network (RBFNN) [40], in which the unknown parameters are estimated by adaptive control method. Moreover, the unmeasured states are estimated by a state observer via output feedback.

(iii) Suitable BLFs are designed to guarantee that the output and partial states of the system can be constrained by time-varying boundary functions. In [33], the time-varying bound functions are assumed to be constants and only FSCs are considered. In [32,35], the authors considered only the case of output constraints, which is a special case of the time-varying PSCs considered in this paper.

The remaining of the work is arranged as below. Preliminaries of algebraical graph theory and RBFNN are introduced in Section 2. In Section 3, the state observer is constructed. In Section 4, the event-triggered controller is designed. In Section 5, the stability analysis is given. In Section 6, a simulation example is given. In Section 7, conclusions are drawn.

Notations : $R^+ = (0, +\infty)$. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, respectively. $\|\cdot\|_2$ denotes the 2-norm and $\|\cdot\|_\infty$ the ∞-norm. $\min\{a_i\}$ and $\max\{a_i\}, i = 1, 2, \cdots, n$, represent the smallest one and the biggest in a_i, respectively. For a matrix $A, A > 0$ means that A is symmetric and positive definite.

2. Preliminaries and problem statement

2.1. Directed graph theory

For a MAS, we usually use a directed graph $\mathcal{G}$ to express the communication relations between agents. Graph $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ consists of a set of agents $\mathcal{V}=\{0,1,2,\cdots,N\}$, a set of directed edges $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $\mathcal{A}=[a_{ij}]\in R^{(N+1)\times(N+1)}$ with $i,j=0,\cdots,N$. If agent i can sense agent j, then $a_{ij}>0$, otherwise $a_{ij}=0$. Let $\mathcal{N}_i=\{j|(j,i)\in\mathcal{E}\}$ be the neighbors set of agent i and $\mathcal{D}=\text{diag}\{d_0,d_1,d_2,\cdots,d_N\}$ be the in-degree matrix of graph $\mathcal{G}$, where $d_i=\Sigma_{j\in\mathcal{N}_i}a_{ij}$. The Laplacian matrix of graph $\mathcal{G}$ is defined as $\mathcal{L}=(l_{ij})_{(N+1)\times(N+1)}=\mathcal{D}-\mathcal{A}$. A graph $\mathcal{G}$ contains a directed spanning tree if there exists at least a directed path from the root to all other agents.

Assumption 1. The graph $\mathcal{G}$ contains a spanning tree with the leader being its root.

The Laplacian $\mathcal{L}$ can be partitioned as

$$\mathcal{L}=\begin{pmatrix}0 & 0_{1\times N}\\ \mathcal{L}_2 & \mathcal{L}_1\end{pmatrix}, \tag{1}$$

where $\mathcal{L}_1\in R^{N\times N}$ and $\mathcal{L}_2\in R^{N\times 1}$.

Lemma 1 [7]. *All nonzero eigenvalues of $\mathcal{L}$ have positive real parts except for zero eigenvalues with 1 as its right eigenvector. Zero eigenvalue is simple if and only if $\mathcal{G}$ contains a spanning tree with leader as the root.*

2.2. System description

Consider the following SMASs:

$$\begin{cases} d\xi_{iq}=(\xi_{i,q+1}+d_{iq}(\xi_i,t)+f_{iq}(\underline{\xi}_{iq}))dt+p_{iq}(\xi_i)d\omega, q=1,2,\cdots,n-1,\\ d\xi_{in}=(\text{sat}_i(u_i(t))+d_{in}(\xi_i,t)+f_{in}(\underline{\xi}_{in}))dt+p_{in}(\xi_i)d\omega,\\ \zeta_i=\xi_{i1}, i=1,2,\cdots,N, \end{cases} \tag{2}$$

where $\underline{\xi}_{iq}=[\xi_{i1},\xi_{i2},\cdots,\xi_{iq}]^T\in R^q, q=1,2,\cdots,n$, is the state vector and $\zeta_i\in R$ is the system output. Let $\xi_i=[\xi_{i1},\cdots,\xi_{in}]^T$ be the full state, which can be divided into two parts. Without loss of generality, we assume that the state $[\xi_{i1},\cdots,\xi_{i\Delta}]^T$ is constrained and the state $[\xi_{i,\Delta+1},\cdots,\xi_{in}]^T$ unconstrained, where $1\leqslant\Delta\leqslant n$. In the special case, when $\Delta=1$, only the output of the system is constrained; when $\Delta=n$, the full states of the system are constrained. The constrained states $\xi_{iq}, q=1,\cdots,\Delta$ for agent i, are restricted to time-varying regions $|\xi_{iq}|<k_{c_{iq}}(t)$, where $k_{c_{iq}}(t)$ is a boundary function. $d_{iq}(\xi_i,t)\in R$ is the external disturbance, which is assumed to be a bounded unknown. $f_{iq}(\cdot)\in R$ and $p_{iq}(\cdot)\in R$ denote unknown and smooth nonlinear functions. $\omega\in R$ is a r-dimensional standard Brownian motion with $E\{d\omega\cdot d\omega^T\}=\sigma\cdot\sigma^T dt$. $\text{sat}_i(u_i(t))\in R$ is the saturated control input, which is defined as

$$\text{sat}_i(u_i(t))=\begin{cases}\text{sign}(u_i(t))u_{im}, & \text{if } |u_i(t)|>u_{im},\\ u_i(t), & \text{if } |u_i(t)|\leqslant u_{im},\end{cases} \tag{3}$$

where u_{im} is a positive constant. It can be transformed into

$$\text{sat}_i(u_i(t))=\chi_i(u_i(t))u_i(t), \tag{4}$$

with $\chi_i(\cdot)$ defined as

$$\chi_i(u_i(t))=\begin{cases}\frac{u_{im}}{u_i(t)}, & \text{if} u_i(t)>u_{im},\\ 1, & \text{if } -u_{im}\leqslant u_i(t)\leqslant u_{im},\\ \frac{-u_{im}}{u_i(t)}, & \text{if } -u_{im}>u_i(t).\end{cases} \tag{5}$$

The function $\chi_i(u_i(t))\in(0,1]$ indicates the degree of the saturation of u_i. In special case of $\chi_i(u_i(t))=1$, that means no saturation occurs. Assume that controller u_i does not go to infinity, which is reasonable for practical applications. Under this assumption, one gets [41]

$$0<l_i^{(1)}\leqslant\min(\chi_i(u_i(t)))\leq 1, \tag{6}$$

where $l_i^{(1)}$ is an unknown constant, which will be estimated later.

Remark 1. According to linear system theory, if a linear system $\dot{x} = \bar{A}x + \bar{B}u$ is controllable, it can be transformed into the Brunovsky controller form, which is the linear part of stochastic system (2). As an extension of general linear systems, stochastic systems (2) can be applied to a large class of realistic systems, such as aircraft, robots, etc.

Substituting (4) into system (2), we have

$$\begin{cases} d\xi_i = (A_i\xi_i + G_i\zeta_i + f_i + d_i + B\chi_i(u_i(t))u_i(t))dt + P_i(\xi_i)d\omega, \\ \zeta_i = C\xi_i, i = 1, 2, \cdots, N, \end{cases} \tag{7}$$

where

$$A_i = \begin{pmatrix} -g_{i1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -g_{i,n-1} & 0 & \cdots & 1 \\ -g_{in} & 0 & \cdots & 0 \end{pmatrix}, G_i = \begin{pmatrix} g_{i1} \\ \vdots \\ g_{i,n-1} \\ g_{in} \end{pmatrix}, P_i(\xi_i) = \begin{pmatrix} p_{i1}(\xi_i) \\ \vdots \\ p_{i,n-1}(\xi_i) \\ p_{in}(\xi_i) \end{pmatrix},$$

$$d_i = \begin{pmatrix} d_{i1} \\ \vdots \\ d_{i,n-1} \\ d_{in} \end{pmatrix}, \xi_i = \begin{pmatrix} \xi_{i1} \\ \vdots \\ \xi_{i,n-1} \\ \xi_{in} \end{pmatrix}, f_i = \begin{pmatrix} f_{i1}(\underline{\xi}_{i1}) \\ \vdots \\ \vdots \\ f_{i,n-1}(\underline{\xi}_{i,n-1}) \\ f_{in}(\underline{\xi}_{in}) \end{pmatrix},$$

$B^T = [0, \cdots, 0, 1]$ and $C = [1, \cdots, 0, 0]$.

In system (7), A_i is Hurwitz by selecting matrix G_i. Thus, for given $Q_i > 0$, there exists $P_i > 0$ such that

$$A_i^T P_i + P_i A_i = -Q_i. \tag{8}$$

Control Objectives: The purpose is to design an ETC machanism for SMASs (1) to achieve the following objectives:

(i) The system outputs $\zeta_i, i = 1, \cdots, N$, can follow the leader's output ζ_0 and tracking errors converge to a small neighborhood of the origin.
(ii) System output and partial state constraints are not violated, ie. $|\xi_{iq}| < k_{c_{iq}}(t), q = 1, 2, \cdots, \Delta, \forall t > 0$.
(iii) All the resulting systems signals are bounded.
(iv) The Zeno behavior [42] can be avoided.

Assumption 2. For the leader signal $\zeta_0(t), \dot{\zeta}_0$ and $\ddot{\zeta}_0$ are continuous and bounded, ie., $|\zeta_0| \leqslant a_0, |\dot{\zeta}_0| \leqslant a_1$ and $|\ddot{\zeta}_0| \leqslant a_2$ with a_0, a_1 and a_2 being constants, $\forall t \geqslant 0$.

Assumption 3. For the unknown and smooth nonlinear function $f_{iq}(\cdot) \in R$, the following inequality

$$|f_{iq}(\underline{\zeta}_{iq}) - f_{iq}(\underline{\xi}_{iq})| \leqslant L_{iq}(|\zeta_{i1} - \xi_{i1}| + \cdots + |\zeta_{iq} - \xi_{iq}|), \tag{9}$$

holds, where L_{iq} are known positive constants and $\underline{\zeta}_{iq} = [\zeta_{i1}, \zeta_{i2}, \cdots, \zeta_{iq}], \zeta_{iq} \in R, q = 1, 2, \cdots, n$.

Assumption 4. For the bounded external disturbance $d_i(\xi, t), |d_i(\xi, t)| \leqslant \bar{d}_i$ with $\bar{d}_i > 0$ being a constant.

Assumption 5. The disturbance covariance $p_i^T \sigma \sigma^T p_i = \bar{\sigma}_i \bar{\sigma}_i^T$ is bounded with $p_i = [p_{i1}, \cdots, p_{in}]^T$.

Lemma 2. [43] For $\forall(q_1, q_2) \in R^2$, the following inequality holds:

$$q_1 q_2 \leqslant \frac{p^{\alpha_1}}{\alpha_1}|q_1|^{\alpha_1} + \frac{1}{\alpha_2 p^{\alpha_2}}|q_2|^{\alpha_2}, \tag{10}$$

where $\alpha_1 > 1, p > 0, \alpha_2 > 1$, and $(\alpha_1 - 1)(\alpha_2 - 1) = 1$.

Lemma 3 [44]. *For any* $\vartheta > 0$ *and* η,

$$0 \leqslant |\eta| - \eta \tanh(\frac{\eta}{\vartheta}) \leq 0.2785\vartheta. \tag{11}$$

2.3. Stochastic Stability

aaa

Definition 1. [45] For stochastic system

$$d\xi = g(\xi)dt + p(\xi)d\omega, \tag{12}$$

where $\xi \in R^n$ is the system state, w is a r-dimension standard Brownian motion. $g(\cdot) \in R^n$ and $p(\cdot) \in R^{n\times r}$ are locally Lipschitz in ξ and satisfy $g(0) = 0$ and $p(0) = 0$. For $\forall f(\xi) \in C^2$, the differential operator $\mathcal{L}$ is defined as:

$$\mathcal{L}f(\xi) = \frac{\partial f}{\partial \xi} g(\xi) + \frac{1}{2} tr\{p^T(\xi)\frac{\partial^2 f}{\partial \xi^2} p(\xi)\}, \tag{13}$$

where tr is the trace of matrix.

Lemma 4 [46]. *There exist C^2 function $f : R^n \to R^+$, class $\mathcal{K}_\infty$ functions g_1, g_2 and constant β_3, β_4, such that $g_1(|\xi|) \leqslant f(\xi) \leqslant g_2(|\xi|)$ and $\mathcal{L}f(\xi) \leqslant -\beta_3 f(\xi) + \beta_4$, for $\forall t > 0$ and $\forall \xi \in R^n$. Further, there exists a unique strong solution of system (12) for each $\xi_0 \in R^n$, such that*

$$E[f(\xi)] \leqslant f(\xi_0)e^{-\beta_3 t} + \frac{\beta_4}{\beta_3}, \forall t > 0. \tag{14}$$

2.4. RBFNN

The unknown nonlinear functions are approximated by RBFNNs [47]:

$$\Gamma(\xi) = \theta^T \varphi(\xi), \tag{15}$$

with l nodes, where $\varphi(\xi) = (\varphi_1(\xi), \cdots, \varphi_l(\xi))^T \in R^l$ is the basis function vector, $\theta = (\theta_1, \cdots, \theta_l)^T \in R^l$ is the weight vector and $\varphi_i(\xi)$ is chosen as

$$\varphi_i(\xi) = \exp[-\frac{(\xi - r_i)^T(\xi - r_i)}{\mu_i^2}], i = 1, 2, \cdots, l, \tag{16}$$

where μ_i is the width of $\varphi_i(\xi)$ and r_i the center.

Using RBFNN, a nonlinear function $f_{iq}(\hat{\underline{\xi}}_{iq})$ can be approximated by

$$\hat{f}_{iq}(\hat{\underline{\xi}}_{iq}|\theta_{iq}) = \theta_{iq}^T \varphi_{iq}(\hat{\underline{\xi}}_{iq}), \tag{17}$$

where $\hat{\underline{\xi}}_{iq} = [\hat{\xi}_{i1}, \hat{\xi}_{i2}, \cdots, \hat{\xi}_{iq}]^T, q = 1, 2, \cdots, n$, is the observer state, which will be designed later. Then, the optimal parameter vector θ_{iq}^* can be expressed as

$$\theta_{iq}^* = \arg \min_{\theta_{iq} \in \Omega} [\sup_{\hat{\underline{\xi}}_{iq} \in \Phi_{iq}} |\hat{f}_{iq}(\hat{\underline{\xi}}_{iq}|\theta_{iq}) - f_{iq}(\hat{\underline{\xi}}_{iq})|], \tag{18}$$

where Ω_{iq} and Φ_{iq} are compact sets corresponding to θ_{iq} and $\hat{\underline{\xi}}_{iq}$, respectively.

Meanwhile, the minimum approximation error $\varepsilon_{iq}(\hat{\underline{\xi}}_{iq})$ is

$$\varepsilon_{iq}(\hat{\underline{\xi}}_{iq}) = f_{iq}(\hat{\underline{\xi}}_{iq}) - \hat{f}_{iq}(\hat{\underline{\xi}}_{iq}|\theta_{iq}^*), \tag{19}$$

which is assumed to be bounded based on neural network approximation theory, that is, $|\varepsilon_{iq}(\hat{\underline{\xi}}_{iq})| \leqslant \bar{\varepsilon}_{iq}$ with $\bar{\varepsilon}_{iq} > 0$ being a constant.

3. Observer design

An observer is constructed to estimate the unmeasurable states as follows:

$$\begin{cases} \dot{\hat{\xi}}_{iq} = \hat{\xi}_{i,q+1} + \theta_{iq}^T \varphi_{iq}(\hat{\underline{\xi}}_{iq}) + g_{iq}(y_i - \hat{\xi}_{i1}), q = 1, 2, \cdots, n-1, \\ \dot{\hat{\xi}}_{in} = \chi_i(u_i(t))u_i(t) + \theta_{in}^T \varphi_{in}(\hat{\underline{\xi}}_i) + g_{in}(y_i - \hat{\xi}_{i1}), \\ \hat{\zeta}_i = \hat{\xi}_{i1}, i = 1, 2, \cdots, N, \end{cases} \tag{20}$$

where $\theta_{iq}, q = 1, \cdots, n$, are the adaptive parameter vectors, $g_{iq}, q = 1, \cdots, n$, are the observer gain parameters.

Then, the matrix form of system (20) is

$$\begin{cases} \dot{\hat{\xi}}_i = A_i \hat{\xi}_i + G_i y_i + \hat{f}_i + B\chi_i(u_i(t))u_i(t), \\ \hat{\zeta}_i = C\hat{\xi}_i, \end{cases} \tag{21}$$

where $\hat{f}_i = [\theta_{i1}^T \varphi_{i1}(\hat{\xi}_{i1}), \cdots, \theta_{in}^T \varphi_{in}(\hat{\underline{\xi}}_i)]^T$.

Let $\tilde{f}_{iq} = f_{iq}(\underline{\xi}_{iq}) - f_{iq}(\hat{\underline{\xi}}_{iq}), q = 1, 2, \cdots, n, \tilde{f}_i = [\tilde{f}_{i1}, \cdots, \tilde{f}_{in}]^T$ and $\tilde{\xi}_i = \xi_i - \hat{\xi}_i = [\tilde{\xi}_{i1}, \tilde{\xi}_{i2}, \cdots, \tilde{\xi}_{in}]^T$ be the observer error. From (13), (7) and (21), one gets

$$d\tilde{\xi}_i = (A_i \tilde{\xi}_i + \tilde{f}_i + \tilde{\theta}_i^T \varphi_i(\hat{\underline{\xi}}_i) + \varepsilon_i + d_i)dt + P_i(\xi)d\omega, \tag{22}$$

where $\tilde{\theta}_{iq} = \theta_{iq}^* - \theta_{iq}, q = 1, 2, \cdots, n, \tilde{\theta}_i = \mathrm{diag}[\tilde{\theta}_{i1}, \tilde{\theta}_{i2}, \cdots, \tilde{\theta}_{in}]$, $\varphi_i(\hat{\underline{\xi}}_i) = [\varphi_{i1}^T(\hat{\underline{\xi}}_{i1}), \varphi_{i2}^T(\hat{\underline{\xi}}_{i2}), \cdots, \varphi_{in}^T(\hat{\underline{\xi}}_{in})]^T$, $\varepsilon_i = [\varepsilon_{i1}, \cdots, \varepsilon_{in}]^T$, $d_i = [d_{i1}, \cdots, d_{in}]^T$.

Remark 2. The unknown nonlinear function f_{iq} in (2) is approximated by RBFNN, and the incremental function term $\tilde{f}_{iq}$ will be dealt with the Lipchitz condition given in Assumption 3.

Construct a Lyapunov function V_0 as follows

$$V_0 = \Sigma_{i=1}^N V_{i0} = \Sigma_{i=1}^N \tilde{\xi}_i^T P_i \tilde{\xi}_i. \tag{23}$$

From (13), (22) and (23), one has

$$\begin{aligned} \mathcal{L}V_{i0} &= 2\tilde{\xi}_i^T P_i \dot{\tilde{\xi}}_i = \tilde{\xi}_i^T (A_i^T P_i + P_i A_i)\tilde{\xi}_i + 2\tilde{\xi}_i^T P_i(\tilde{\theta}_i^T \varphi_i(\hat{\underline{\xi}}_i) + \tilde{f}_i + \varepsilon_i + d_i) + tr(\sigma p_i^T P_i p_i \sigma^T) \\ &\leqslant -\lambda_{\min}(Q_i)\|\tilde{\xi}_i\|^2 + 2\tilde{\xi}_i^T P_i(\tilde{\theta}_i^T \varphi_i(\hat{\underline{\xi}}_i) + \tilde{f}_i + \varepsilon_i + d_i) + tr(\sigma p_i^T P_i p_i \sigma^T). \end{aligned} \tag{24}$$

Note that $\varphi_i(\hat{\underline{\xi}}_i)\varphi_i^T(\hat{\underline{\xi}}_{iq}) \le 1$. Under Assumptions 3–5 and from Lemma 2, one has

$$2\tilde{\xi}_i^T P_i \tilde{f}_i \le 2(\tilde{\xi}_i^T P_i \tilde{\xi}_i)^{\frac{1}{2}} (\tilde{f}_i^T P_i \tilde{f})^{\frac{1}{2}} \le 2\|\tilde{\xi}_i\|^2 \|P_i\| \Sigma_{q=1}^n q L_{iq}, \tag{25}$$

$$2\tilde{\xi}_i^T P_i \tilde{\theta}_i^T \varphi_i(\hat{\underline{\xi}}_i) \leqslant \lambda_{\max}^2(P_i)\|\tilde{\xi}_i\|^2 + \Sigma_{q=1}^n \tilde{\theta}_{iq}^T \tilde{\theta}_{iq}, \tag{26}$$

$$2\tilde{\xi}_i^T P_i(\varepsilon_i + d_i) \le 2\|\tilde{\xi}_i\|^2 + \|P_i\|^2\|\bar{\varepsilon}_i\|^2 + \|P_i\|^2\|\bar{d}_i\|^2, \tag{27}$$

and

$$tr(\sigma p_i^T P_i p_i \sigma^T) \leqslant \frac{1}{2}\|P_i\|^2 + \frac{1}{2}|\bar{\sigma}_i \bar{\sigma}_i^T|^2, \tag{28}$$

where $\bar{\varepsilon}_i = [\bar{\varepsilon}_{i1}, \cdots, \bar{\varepsilon}_{in}]^T, \bar{d}_i = [\bar{d}_{i1}, \cdots, \bar{d}_{i,n-1}, \bar{d}_{in}]^T$.

Substituting (25)-(28) into (24), one has

$$\begin{aligned} \mathcal{L}V_{i0} \leqslant{}& -\lambda_{\min}(Q_i)\|\tilde{\xi}_i\|^2 + \lambda_{\max}^2(P_i)\|\tilde{\xi}_i\|^2 + \Sigma_{q=1}^n \tilde{\theta}_{iq}^T \tilde{\theta}_{iq} + \|P_i\|^2\|\bar{\varepsilon}_i\|^2 + 2\|\tilde{\xi}_i\|^2 + \|P_i\|^2\|\bar{d}_i\|^2 + 2\|\tilde{\xi}_i\|^2\|P_i\|\Sigma_{q=1}^n q L_{iq} \\ &+ \frac{1}{2}\|P_i\|^2 + \frac{1}{2}|\bar{\sigma}_i \bar{\sigma}_i^T|^2. \end{aligned} \tag{29}$$

From (23) and (29), one gets

$$\begin{aligned} \mathcal{L}V_0 ={}& \mathcal{L}(\Sigma_{i=1}^N V_{i0}) \\ \leqslant{}& \Sigma_{i=1}^N[-(\lambda_{\min}(Q_i) - 2 - 2\|P_i\|\Sigma_{q=1}^n q L_{iq} - \lambda_{\max}^2(P_i))\|\tilde{\xi}_i\|^2 \\ &+ M_i^{(1)} + \Sigma_{q=1}^n \tilde{\theta}_{iq}^T \tilde{\theta}_{iq}], \end{aligned} \tag{30}$$

where $M_i^{(1)} = \frac{1}{2}\|P_i\|^2 + \frac{1}{2}|\bar{\sigma}_i \bar{\sigma}_i^T|^2 + \|P_i\|^2\|\bar{\varepsilon}_i\|^2 + \|P_i\|^2\|\bar{d}_i\|^2$.

4. Event-triggered controller design

An adaptive ETC scheme is designed in this section to solve the control problem considered in this paper.

To handle the PSCs, a BLF candidate is used for control design. Let us define

$$V(t) = \frac{1}{2}\log\frac{\kappa^4(t)}{\kappa^4(t) - \eta^4(t)}, \tag{31}$$

where log is the natural logarithm, $\eta(t)$ is constrained by $|\eta(t)| < \kappa(t)$ with $\kappa(t) > 0$ being a boundary constraint function.

Remark 3. Generally, a BLF is chosen as $\frac{1}{2}\log\frac{\kappa^2(t)}{\kappa^2(t)-\eta^2(t)}$. However, for stochastic systems, since the differential operator $\mathcal{L}$ defined in (13) contains the second derivative, a BLF defined in (31) is adopted to avoid the error term appearing in the denominator of the proposed controller.

Lemma 5. [46] For any given $\kappa(t) > 0$ and all $\eta(t)$ satisfying $|\eta(t)| < \kappa(t)$, the following inequality $\log\frac{\kappa^{2s}(t)}{\kappa^{2s}(t)-\eta^{2s}(t)} < \frac{\eta^{2s}(t)}{\kappa^{2s}(t)-\eta^{2s}(t)}$ holds, where s is a positive integer.

Let

$$\eta_{i1} = \Sigma_{j=1}^{N} a_{ij}(\zeta_i - \zeta_j) + a_{i0}(\zeta_i - \zeta_0), \tag{32}$$

$$\eta_{iq} = \hat{\xi}_{iq} - \beta_{iq}, \tag{33}$$

and

$$z_{iq} = \beta_{iq} - \alpha_{i,q-1}, q = 2, \cdots, n, \tag{34}$$

where η_{i1} is the local consensus error, ζ_0 is the leader signal, α_{iq} and $\eta_{in}, q = 2, \cdots, n$, are the intermediate control functions and the error surfaces, respectively, and $\beta_{iq}, q = 2, \cdots, n$, is the filter output of the first-order filter, which is defined as

$$\varpi_{iq}\dot{\beta}_{iq} + \beta_{iq} = \alpha_{i,q-1}, \beta_{iq}(0) = \alpha_{i,q-1}(0), \tag{35}$$

where $\varpi_{iq} > 0$ is a constant.

Remark 4. In order to design a distributed controller, the local consensus error η_{i1} defined in (32) is introduced instead of $\zeta_i - \zeta_0$ to avoid requiring global information all the time.

Remark 5. Similar to [48], a first-order filter (35) is introduced. Compare with the traditional backstepping method, the problem of complexity explosion resulting from repeatedly differentiating virtual control signal $\alpha_{i,q-1}$ can be avoided.

Step 1: From (2), (17)–(19) and (32)–(34), one has

$$\begin{aligned} d\eta_{i1} = & [\mathscr{d}_i\dot{\zeta}_i - \Sigma_{j=1}^{N} a_{ij}\dot{\zeta}_j - a_{i0}\dot{\zeta}_0]dt \\ = & \mathscr{d}_i(\eta_{i2} + z_{i2} + \alpha_{i1} + \tilde{\xi}_{i2} + \tilde{\theta}_{i1}^T\varphi_{i1}(\hat{\xi}_{i1}) + \theta_{i1}^T\varphi_{i1}(\hat{\xi}_{i1}) \\ & + \varepsilon_{i1} + d_{i1})dt + (\mathscr{d}_i p_{i1} - \Sigma_{j=1}^{N} a_{ij}p_{j1})d\omega - \Sigma_{j=1}^{N} a_{ij}(\hat{\xi}_{j2} \\ & + \tilde{\xi}_{j2} + \varepsilon_{j1} + \tilde{\theta}_{j1}^T\varphi_{j1}(\hat{\xi}_{j1}) + \theta_{j1}^T\varphi_{j1}(\hat{\xi}_{j1}) + d_{j1})dt - a_{i0}\dot{\zeta}_0 dt. \end{aligned} \tag{36}$$

Choose the following Lyapunov function

$$\begin{aligned} V_1 = & V_0 + \Sigma_{i=1}^{N} V_{i1} \\ = & V_0 + \Sigma_{i=1}^{N}\left(\frac{1}{4}\log\frac{\kappa_{i1}^4(t)}{\kappa_{i1}^4(t)-\eta_{i1}^4} + \frac{1}{2c_{i1}^{(1)}}\tilde{\theta}_{i1}^T\tilde{\theta}_{i1} + \Sigma_{j=1}^{N}\frac{a_{ij}}{2c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\tilde{\theta}_{j1}\right), \end{aligned} \tag{37}$$

where $\tilde{\theta}_{i1} = \theta_{i1}^* - \theta_{i1}, c_{i1}^{(1)}$ and $c_{j1}^{(2)}$ are positive design parameters. $\kappa_{i1}(t) > 0$ is a boundary constraint function satisfying $|\eta_{i1}| < \kappa_{i1}(t)$ and will be given later.

From (13), (36) and (37), one gets

$$
\begin{aligned}
\mathcal{L}V_{i1} = & \frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}[\mathscr{d}_i(\eta_{i2}+\alpha_{i1}+\tilde{\xi}_{i2}+\theta_{i1}^T\varphi_{i1}(\hat{\xi}_{i1})+\varepsilon_{i1}+d_{i1}+\tilde{\theta}_{i1}^T\varphi_{i1}(\hat{\xi}_{i1})) \\
& -\Sigma_{j=1}^N a_{ij}(\hat{\xi}_{j2}+\tilde{\xi}_{j2}+\varepsilon_{j1}+\theta_{j1}^T\varphi_{j1}(\hat{\xi}_{j1})+\tilde{\theta}_{j1}^T\varphi_{j1}(\hat{\xi}_{j1})+d_{j1})-a_{i0}\dot{\zeta}_0] \\
& -\frac{\eta_{i1}^4\dot{\kappa}_{i1}}{\kappa_{i1}(\kappa_{i1}^4-\eta_{i1}^4)}+\frac{1}{2}(\frac{3\eta_{i1}^2}{\kappa_{i1}^4-\eta_{i1}^4}-\frac{4\eta_{i1}^3(\kappa_{i1}^3-\eta_{i1}^3)}{(\kappa_{i1}^4-\eta_{i1}^4)^2}-\frac{4\eta_{i1}^3}{\kappa_{i1}(\kappa_{i1}^4-\eta_{i1}^4)} \\
& +\frac{\eta_{i1}^4}{\kappa_{i1}^2(\kappa_{i1}^4-\eta_{i1}^4)}+\frac{4\eta_{i1}^4(\kappa_{i1}^3-\eta_{i1}^3)}{\kappa_{i1}^2(\kappa_{i1}^4-\eta_{i1}^4)^2})[\mathscr{d}_i^2 p_{i1}^T\sigma\sigma^T p_{i1} \\
& -\Sigma_{j=1}^N a_{ij}\Sigma_{j=1}^N a_{ij}p_{j1}^T\sigma\sigma^T p_{j1}]+\frac{1}{c_{i1}^{(1)}}\tilde{\theta}_{i1}^T\dot{\tilde{\theta}}_{i1}+\Sigma_{j=1}^N\frac{a_{ij}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\dot{\tilde{\theta}}_{j1}.
\end{aligned}
\tag{38}
$$

Under Assumptions 3–5 and from Lemma 2, one has

$$
\mathscr{d}_i\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}(\tilde{\xi}_{i2}+d_{i1}+\varepsilon_{i1})\leqslant\frac{3}{2}\mathscr{d}_i^2(\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4})^2+\frac{1}{2}\|\tilde{\xi}_i\|^2+\frac{1}{2}\bar{d}_{i1}^2+\frac{1}{2}\bar{\varepsilon}_{i1}^2,
\tag{39}
$$

$$
\begin{aligned}
& -\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}\Sigma_{j=1}^N a_{ij}(\tilde{\xi}_{j2}+d_{j1}+\varepsilon_{j1}) \\
\leqslant & \frac{3}{2}(\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4})^2+\frac{1}{2}\|\mathcal{L}_1\tilde{\xi}\|^2+\frac{1}{2}(\Sigma_{j=1}^N a_{ij}\bar{d}_{j1})^2+\frac{1}{2}(\Sigma_{j=1}^N a_{ij}\bar{\varepsilon}_{j1})^2 \\
\leqslant & \frac{3}{2}(\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4})^2+\frac{1}{2}\|\mathcal{L}_1\|^2\|\tilde{\xi}\|^2+\frac{1}{2}(\Sigma_{j=1}^N a_{ij}\bar{d}_{j1})^2+\frac{1}{2}(\Sigma_{j=1}^N a_{ij}\bar{\varepsilon}_{j1})^2,
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
& \frac{1}{2}(\frac{3\eta_{i1}^2}{\kappa_{i1}^4-\eta_{i1}^4}-\frac{4\eta_{i1}^3(\kappa_{i1}^3-\eta_{i1}^3)}{(\kappa_{i1}^4-\eta_{i1}^4)^2}-\frac{4\eta_{i1}^3}{\kappa_{i1}(\kappa_{i1}^4-\eta_{i1}^4)}+\frac{\eta_{i1}^4}{\kappa_{i1}^2(\kappa_{i1}^4-\eta_{i1}^4)} \\
& +\frac{4\eta_{i1}^4(\kappa_{i1}^3-\eta_{i1}^3)}{\kappa_{i1}^2(\kappa_{i1}^4-\eta_{i1}^4)^2})\mathscr{d}_i^2 p_{i1}^T\sigma\sigma^T p_{i1} \\
\leqslant & \frac{1}{2}(\frac{9\eta_{i1}^4}{(\kappa_{i1}^4-\eta_{i1}^4)^2}+\frac{16\eta_{i1}^6(\kappa_{i1}^3-\eta_{i1}^3)^2}{(\kappa_{i1}^4-\eta_{i1}^4)^4}+\frac{16\eta_{i1}^6}{\kappa_{i1}^2(\kappa_{i1}^4-\eta_{i1}^4)^2}+\frac{\eta_{i1}^8}{\kappa_{i1}^4(\kappa_{i1}^4-\eta_{i1}^4)^2} \\
& +\frac{16\eta_{i1}^8(\kappa_{i1}^3-\eta_{i1}^3)^2}{\kappa_{i1}^4(\kappa_{i1}^4-\eta_{i1}^4)^4})+\frac{5}{4}|\bar{\sigma}_i\bar{\sigma}_i^T|^2\mathscr{d}_i^4,
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
& \frac{1}{2}(\frac{3\eta_{i1}^2}{\kappa_{i1}^4-\eta_{i1}^4}-\frac{4\eta_{i1}^3(\kappa_{i1}^3-\eta_{i1}^3)}{(\kappa_{i1}^4-\eta_{i1}^4)^2}-\frac{4\eta_{i1}^3}{\kappa_{i1}(\kappa_{i1}^4-\eta_{i1}^4)}+\frac{\eta_{i1}^4}{\kappa_{i1}^2(\kappa_{i1}^4-\eta_{i1}^4)} \\
& +\frac{4\eta_{i1}^4(\kappa_{i1}^3-\eta_{i1}^3)}{\kappa_{i1}^2(\kappa_{i1}^4-\eta_{i1}^4)^2})\Sigma_{j=1}^N a_{ij}\Sigma_{j=1}^N a_{ij}p_{j1}^T\sigma\sigma^T p_{j1} \\
\leqslant & \frac{1}{2}(\frac{9\eta_{i1}^4}{(\kappa_{i1}^4-\eta_{i1}^4)^2}+\frac{16\eta_{i1}^6(\kappa_{i1}^3-\eta_{i1}^3)^2}{(\kappa_{i1}^4-\eta_{i1}^4)^4}+\frac{16\eta_{i1}^6}{\kappa_{i1}^2(\kappa_{i1}^4-\eta_{i1}^4)^2}+\frac{\eta_{i1}^8}{\kappa_{i1}^4(\kappa_{i1}^4-\eta_{i1}^4)^2} \\
& +\frac{16\eta_{i1}^8(\kappa_{i1}^3-\eta_{i1}^3)^2}{\kappa_{i1}^4(\kappa_{i1}^4-\eta_{i1}^4)^4})+\frac{5}{4}(\Sigma_{j=1}^N a_{ij}\Sigma_{j=1}^N a_{ij}|\bar{\sigma}_j\bar{\sigma}_j^T|)^2,
\end{aligned}
\tag{42}
$$

and

$$
\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}\mathscr{d}_i(\eta_{i2}+z_{i2})\leqslant\frac{6}{4}\mathscr{d}_i^{\frac{4}{3}}(\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4})^{\frac{4}{3}}+\frac{1}{4}\eta_{i2}^4+\frac{1}{4}z_{i2}^4.
\tag{43}
$$

Substituting (39)-(43) into (38), one obtains that

$$\begin{aligned}
\mathcal{L}V_{i1} \leqslant\ & \frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}[\mathscr{d}_i(\alpha_{i1}+\theta_{i1}^T\varphi_{i1}(\hat{\xi}_{i1}))-\frac{\eta_{i1}\dot{\kappa}_{i1}}{\kappa_{i1}}-a_{i0}\dot{\zeta}_0-\Sigma_{j=1}^N a_{ij}(\hat{\xi}_{j2}\\
&+\theta_{j1}^T\varphi_{j1}(\hat{\xi}_{j1}))]+\frac{1}{2}\bar{d}_{i1}^2+\frac{1}{2}\bar{\varepsilon}_{i1}^2+\frac{1}{2}\|\tilde{\xi}_i\|^2+\frac{1}{4}\eta_{i2}^4+\frac{3}{2}\mathscr{d}_i^2\left(\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}\right)^2\\
&+\frac{1}{4}z_{i2}^4+\frac{3}{2}\left(\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}\right)^2+\frac{6}{4}\mathscr{d}_i^{\frac{4}{3}}\left(\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}\right)^{\frac{4}{3}}+\frac{1}{2}\left(\Sigma_{j=1}^N a_{ij}\bar{d}_{j1}\right)^2\\
&+\frac{1}{2}\left(\Sigma_{j=1}^N a_{ij}\bar{\varepsilon}_{j1}\right)^2+\frac{5}{4}|\bar{\sigma}_i\bar{\sigma}_i^T|^2\mathscr{d}_i^4+\frac{5}{4}\left(\Sigma_{j=1}^N a_{ij}\Sigma_{j=1}^N a_{ij}|\bar{\sigma}_j\bar{\sigma}_j^T|\right)^2\\
&+\frac{9\eta_{i1}^4}{(\kappa_{i1}^4-\eta_{i1}^4)^2}+\frac{16\eta_{i1}^6(\kappa_{i1}^3-\eta_{i1}^3)^2}{(\kappa_{i1}^4-\eta_{i1}^4)^4}+\frac{16\eta_{i1}^6}{\kappa_{i1}^2(\kappa_{i1}^4-\eta_{i1}^4)^2}\\
&+\frac{\eta_{i1}^8}{\kappa_{i1}^4(\kappa_{i1}^4-\eta_{i1}^4)^2}+\frac{16\eta_{i1}^8(\kappa_{i1}^3-\eta_{i1}^3)^2}{\kappa_{i1}^4(\kappa_{i1}^4-\eta_{i1}^4)^4}+\frac{1}{2}\|\mathcal{L}_1\|^2\|\tilde{\xi}\|^2\\
&+\frac{1}{c_{i1}^{(1)}}\tilde{\theta}_{i1}^T\left(\mathscr{d}_i\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}c_{i1}^{(1)}\varphi_{i1}(\hat{\xi}_{i1})-\dot{\theta}_{i1}\right)\\
&+\Sigma_{j=1}^N\frac{a_{ij}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\left(-\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}c_{j1}^{(2)}\varphi_{j1}(\hat{\xi}_{j1})-\dot{\theta}_{j1}\right).
\end{aligned} \tag{44}$$

Construct the virtual controller α_{i1} and the adaptive laws $\dot{\theta}_{i1}$ and $\dot{\theta}_{j1}$ as

$$\begin{aligned}
\alpha_{i1} =\ & -\frac{c_{i1}^{(3)}\eta_{i1}}{\mathscr{d}_i}-\theta_{i1}^T\varphi_{i1}(\hat{\xi}_{i1})+\frac{\eta_{i1}\dot{\kappa}_{i1}}{\mathscr{d}_i\kappa_{i1}}+\frac{1}{\mathscr{d}_i}\left(\Sigma_{j=1}^N a_{ij}(\hat{\xi}_{j2}+\theta_{j1}^T\varphi_{j1}(\hat{\xi}_{j1}))\right)\\
&+\frac{b_i\dot{\zeta}_0}{\mathscr{d}_i}-\frac{6}{4}\mathscr{d}_i^{\frac{1}{3}}\left(\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}\right)^{\frac{1}{3}}-\frac{3}{2}\frac{\mathscr{d}_i\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}-\frac{3\eta_{i1}^3}{2\mathscr{d}_i(\kappa_{i1}^4-\eta_{i1}^4)}\\
&-\frac{9\eta_{i1}}{\mathscr{d}_i(\kappa_{i1}^4-\eta_{i1}^4)}-\frac{16\eta_{i1}^3(\kappa_{i1}^3-\eta_{i1}^3)^2}{\mathscr{d}_i(\kappa_{i1}^4-\eta_{i1}^4)^3}-\frac{16\eta_{i1}^3}{\mathscr{d}_i\kappa_{i1}^2(\kappa_{i1}^4-\eta_{i1}^4)}\\
&-\frac{\eta_{i1}^5}{\mathscr{d}_i\kappa_{i1}^4(\kappa_{i1}^4-\eta_{i1}^4)}-\frac{16\eta_{i1}^5(\kappa_{i1}^3-\eta_{i1}^3)^2}{\mathscr{d}_i\kappa_{i1}^4(\kappa_{i1}^4-\eta_{i1}^4)^3},
\end{aligned} \tag{45}$$

$$\dot{\theta}_{i1} = \mathscr{d}_i\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}c_{i1}^{(1)}\varphi_{i1}(\hat{\xi}_{i1})-c_{i1}^{(4)}\theta_{i1}, \tag{46}$$

and

$$\dot{\theta}_{j1} = -\frac{\eta_{i1}^3}{\kappa_{i1}^4-\eta_{i1}^4}c_{j1}^{(2)}\varphi_{j1}(\hat{\xi}_{j1})-c_{j1}^{(5)}\theta_{j1}, \tag{47}$$

where $c_{i1}^{(3)}>0, c_{i1}^{(4)}>0$ and $c_{j1}^{(5)}>0$ are constants.

Substituting (45)-(47) into (44), one has

$$\begin{aligned}
\mathcal{L}V_{i1} \leqslant\ & -\frac{c_{i1}^{(3)}\eta_{i1}^4}{\kappa_{i1}^4-\eta_{i1}^4}+\frac{1}{4}\eta_{i2}^4+\frac{1}{2}\bar{d}_{i1}^2+\frac{1}{2}\bar{\varepsilon}_{i1}^2+\frac{1}{2}\|\tilde{\xi}_i\|^2+\frac{1}{4}z_{i2}^4+\frac{5}{4}|\bar{\sigma}_i\bar{\sigma}_i^T|^2\mathscr{d}_i^4\\
&+\frac{1}{2}\|\mathcal{L}_1\|^2\|\tilde{\xi}\|^2+\frac{1}{2}\left(\Sigma_{j=1}^N a_{ij}\bar{d}_{j1}\right)^2+\frac{5}{4}\left(\Sigma_{j=1}^N a_{ij}\Sigma_{j=1}^N a_{ij}|\bar{\sigma}_j\bar{\sigma}_j^T|\right)^2\\
&+\frac{1}{2}\left(\Sigma_{j=1}^N a_{ij}\bar{\varepsilon}_{j1}\right)^2+\frac{c_{i1}^{(4)}}{c_{i1}^{(1)}}\tilde{\theta}_{i1}^T\theta_{i1}+\Sigma_{j=1}^N a_{ij}\frac{c_{j1}^{(5)}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\theta_{j1}.
\end{aligned} \tag{48}$$

From (37) and (48), we have

$$\begin{aligned}
\mathcal{L}V_1 =\ & \mathcal{L}[V_0+\Sigma_{i=1}^N V_{i1}]\\
\leqslant\ & \Sigma_{i=1}^N[-M_i^{(2)}\|\tilde{\xi}_i\|^2-\frac{c_{i1}^{(3)}\eta_{i1}^4}{\kappa_{i1}^4-\eta_{i1}^4}+M_i^{(3)}+\frac{1}{4}\eta_{i2}^4+\frac{1}{4}z_{i2}^4+\frac{c_{i1}^{(4)}}{c_{i1}^{(1)}}\tilde{\theta}_{i1}^T\theta_{i1}\\
&+\Sigma_{j=1}^N a_{ij}\frac{c_{j1}^{(5)}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\theta_{j1}+\Sigma_{q=1}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}],
\end{aligned} \tag{49}$$

where $M_i^{(2)}=\lambda_{\min}(Q_i)-\frac{5}{2}-2\|P\|\Sigma_{q=1}^n qL_{iq}-\lambda_{\max}^2(P)-\frac{1}{2}\|\mathcal{L}_1\|^2>0$, $M_i^{(3)}=M_i^{(1)}+\frac{1}{2}\bar{d}_{i1}^2+\frac{1}{2}\bar{\varepsilon}_{i1}^2+\frac{5}{4}|\bar{\sigma}_i\bar{\sigma}_i^T|^2\mathscr{d}_i^4+\frac{1}{2}\left(\Sigma_{j=1}^N a_{ij}\bar{d}_{j1}\right)^2+\frac{1}{2}\left(\Sigma_{j=1}^N a_{ij}\bar{\varepsilon}_{j1}\right)^2+\frac{5}{4}\left(\Sigma_{j=1}^N a_{ij}\Sigma_{j=1}^N a_{ij}|\bar{\sigma}_j\bar{\sigma}_j^T|\right)^2$.

Step 2: From (20), (33) and (34), we have

$$\begin{aligned}\dot{\eta}_{i2} &= \dot{\hat{\xi}}_{i2} - \dot{\beta}_{i2} \\ &= \eta_{i3} + z_{i3} + \alpha_{i2} + \theta_{i2}^T \varphi_{i2}(\hat{\underline{\xi}}_{i2}) + g_{i2}(\zeta_i - \hat{\xi}_{i2}) - \dot{\beta}_{i2}.\end{aligned} \tag{50}$$

From (34) and (35), one has

$$dz_{il} = d\beta_{il} - d\alpha_{i,l-1} = [-\frac{z_{il}}{\varpi_{il}} + \psi_{il}(\cdot)]dt + \phi_{il}(\cdot)d\omega, l = 2, \cdots, n, \tag{51}$$

where ψ_{il} and ϕ_{il} are bounded continuous functions. Thus, there exist $\bar{\psi}_{il} > 0$ and $\bar{\phi}_{il} > 0$ such that $|\psi_{il}| \leqslant \bar{\psi}_{il}, |\phi_{il}| \leqslant \bar{\phi}_{il}$.

Construct the Lyapunov function V_2 as

$$V_2 = V_1 + \Sigma_{i=1}^N V_{2i} = V_1 + \Sigma_{i=1}^N (\frac{1}{4} \log \frac{\kappa_{i2}^4(t)}{\kappa_{i2}^4(t) - \eta_{i2}^4} + \frac{1}{2c_{i2}^{(1)}} \tilde{\theta}_{i2}^T \tilde{\theta}_{i2} + \frac{1}{4} z_{i2}^4), \tag{52}$$

where $\tilde{\theta}_{i2} = \theta_{i2}^* - \theta_{i2}, |\eta_{i2}| < \kappa_{i2}(t)$ and $c_{i2}^{(1)}$ is a positive design parameter.

From (13), (50), (51) and (52), one has

$$\begin{aligned}\mathcal{L}V_{2i} = & \tfrac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4} [(\eta_{i3} + \tilde{\theta}_{i2}^T \varphi_{i2}(\hat{\underline{\xi}}_{i2}) + z_{i3} + \alpha_{i2} - \tilde{\theta}_{i2}^T \varphi_{i2}(\hat{\underline{\xi}}_{i2}) \\ & + \theta_{i2}^T \varphi_{i2}(\hat{\xi}_{i2}) + g_{i2}(\zeta_i - \hat{\xi}_{i2}) - \dot{\beta}_{i2}] - \tfrac{\eta_{i2}^4 \dot{\kappa}_{i2}}{\kappa_{i2}(\kappa_{i2}^4 - \eta_{i2}^4)} \\ & + \tfrac{1}{c_{i2}^{(1)}} \tilde{\theta}_{i2}^T \dot{\tilde{\theta}}_{i2} + z_{i2}^3(-\tfrac{z_{i2}}{\varpi_{i2}} + \psi_{i2}) + \tfrac{3}{2} z_{i2}^2 tr(\phi_{i2}^T \phi_{i2}).\end{aligned} \tag{53}$$

By Lemma 2 and $\varphi_{i2}(\hat{\underline{\xi}}_{i2})\varphi_{i2}^T(\hat{\underline{\xi}}_{i2}) \leq 1$, one has

$$-\frac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4} \tilde{\theta}_{i2}^T \varphi_{i2}(\hat{\underline{\xi}}_{i2}) \leqslant \frac{1}{2} (\frac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4})^2 + \frac{1}{2} \tilde{\theta}_{i2}^T \tilde{\theta}_{i2}, \tag{54}$$

$$\frac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4} (\eta_{i3} + z_{i3}) \leqslant \frac{6}{4} (\frac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4})^{\frac{4}{3}} + \frac{1}{4} z_{i3}^4 + \frac{1}{4} \eta_{i3}^4, \tag{55}$$

$$z_{i2}^3 \psi_{i2} \leqslant \frac{3}{4} \bar{\psi}_{i2}^{\frac{4}{3}} \gamma_{i2}^{\frac{4}{3}} z_{i2}^4 + \frac{1}{4\gamma_{i2}^4}, \tag{56}$$

and

$$\frac{3}{2} z_{i2}^2 tr(\phi_{i2}^T \phi_{i2}) \leqslant \frac{3}{4} \iota_{i2}^2 + \frac{3}{4\iota_{i2}^2} \bar{\phi}_{i2}^4 z_{i2}^4, \tag{57}$$

where γ_{i2} and ι_{i2} are the positive constants.

Substituting (54)-(57) into (53), one gets

$$\begin{aligned}\mathcal{L}V_{i2} \leqslant & \tfrac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4} (\alpha_{i2} + \theta_{i2}^T \varphi_{i2}(\hat{\underline{\xi}}_{i2}) + g_{i2}(\zeta_i - \hat{\xi}_{i1}) - \dot{\beta}_{i2} - \tfrac{\eta_{i2} \dot{\kappa}_{i2}}{\kappa_{i2}}) \\ & + \tfrac{1}{4} \eta_{i3}^4 + \tfrac{1}{4} z_{i3}^4 + \tfrac{1}{2} (\tfrac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4})^2 + \tfrac{1}{2} \tilde{\theta}_{i2}^T \tilde{\theta}_{i2} + \tfrac{1}{4\gamma_{i2}^4} + \tfrac{3}{4} \iota_{i2}^2 \\ & + \tfrac{6}{4} (\tfrac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4})^{\frac{4}{3}} + \tfrac{1}{c_{i2}^{(1)}} \tilde{\theta}_{i2}^T (\tfrac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4} c_{i2}^{(1)} \varphi_{i2}(\hat{\underline{\xi}}_{i2}) - \dot{\theta}_{i2}) \\ & + (\tfrac{3}{4} \bar{\psi}_{i2}^{\frac{4}{3}} \gamma_{i2}^{\frac{4}{3}} - \tfrac{1}{\varpi_{i2}} + \tfrac{3}{4\iota_{i2}^2} \bar{\phi}_{i2}^4) z_{i2}^4.\end{aligned} \tag{58}$$

Construct the virtual controller α_{i2} and the adaptive law $\dot{\theta}_{i2}$ as

$$\begin{aligned}\alpha_{i2} = & -c_{i2}^{(3)} \eta_{i2} - g_{i2}(\zeta_i - \hat{\xi}_{i1}) - \theta_{i2}^T \varphi_{i2}(\hat{\underline{\xi}}_{i2}) - \tfrac{\eta_{i2}^3}{2(\kappa_{i2}^4 - \eta_{i2}^4)} \\ & + \tfrac{\eta_{i2} \dot{\kappa}_{i2}}{\kappa_{i2}} - \tfrac{6}{4} (\tfrac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4})^{\frac{1}{3}} - \tfrac{\eta_{i2}(\kappa_{i2}^4 - \eta_{i2}^4)}{4} + \dot{\beta}_{i2},\end{aligned} \tag{59}$$

and

$$\dot{\theta}_{i2} = \frac{\eta_{i2}^3}{\kappa_{i2}^4 - \eta_{i2}^4} c_{i2}^{(1)} \varphi_{i2}(\underline{\hat{\xi}}_{i2}) - c_{i2}^{(4)} \theta_{i2}, \tag{60}$$

where $c_{i2}^{(3)} > 0$ and $c_{i2}^{(4)} > 0$ are constants.

Substituting (59)-(60) into (58), it is clear that

$$\begin{aligned} \mathcal{L}V_{i2} \leqslant \; & -\tfrac{c_{i2}^{(3)}\eta_{i2}^4}{\kappa_{i2}^4-\eta_{i2}^4} - \tfrac{1}{4}\eta_{i2}^4 + \tfrac{1}{4}\eta_{i3}^4 + \tfrac{1}{4}z_{i3}^4 + \tfrac{1}{2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2} + \tfrac{c_{i2}^{(4)}}{c_{i2}^{(1)}}\tilde{\theta}_{i2}^T\theta_{i2} \\ & + \tfrac{1}{4\gamma_{i2}^4} + \tfrac{3}{4}\imath_{i2}^2 + (\tfrac{3}{4}\bar{\psi}_{i2}^{\frac{4}{3}}\gamma_{i2}^{\frac{4}{3}} - \tfrac{1}{\varpi_{i2}} + \tfrac{3}{4\imath_{i2}^2}\bar{\phi}_{i2}^4)z_{i2}^4. \end{aligned} \tag{61}$$

From (52) and (61), we have

$$\begin{aligned} \mathcal{L}V_2 = \; & \mathcal{L}[V_1 + \Sigma_{i=1}^N V_{i2}] \\ \leqslant \; & \Sigma_{i=1}^N[-M_i^{(2)}\|\tilde{\xi}_i\|^2 - \Sigma_{q=1}^2 \tfrac{c_{iq}^{(3)}\eta_{iq}^4}{\kappa_{iq}^4-\eta_{iq}^4} + M_i^{(3)} + \Sigma_{q=1}^2 \tfrac{c_{iq}^{(4)}}{c_{iq}^{(1)}}\tilde{\theta}_{iq}^T\theta_{iq} \\ & + \tfrac{1}{2}\tilde{\theta}_{i2}^T\tilde{\theta}_{i2} + \tfrac{1}{4}\Sigma_{q=2}^3 z_{iq}^4 + \Sigma_{j=1}^N a_{ij}\tfrac{c_{j1}^{(5)}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\theta_{j1} + \Sigma_{q=1}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} \\ & + \tfrac{1}{4}\eta_{i3}^4 + \tfrac{1}{4\gamma_{i2}^4} + \tfrac{3}{4}\imath_{i2}^2 + (\tfrac{3}{4}\bar{\psi}_{i2}^{\frac{4}{3}}\gamma_{i2}^{\frac{4}{3}} - \tfrac{1}{\varpi_{i2}} + \tfrac{3}{4\imath_{i2}^2}\bar{\phi}_{i2}^4)z_{i2}^4]. \end{aligned} \tag{62}$$

Step $b, b = 3, \cdots, \Delta$: From (20), (33) and (34), we have

$$\dot{\eta}_{ib} = \eta_{i,b+1} + z_{i,b+1} + \alpha_{ib} + \theta_{ib}^T\varphi_{ib}(\underline{\hat{\xi}}_{ib}) + g_{ib}(\zeta_i - \hat{\xi}_{i1}) - \dot{\beta}_{ib}. \tag{63}$$

Construct the Lyapunov function V_b as

$$V_b = V_{b-1} + \Sigma_{i=1}^N V_{bi} = V_{b-1} + \Sigma_{i=1}^N(\frac{1}{4}\log\frac{\kappa_{ib}^4}{\kappa_{ib}^4 - \eta_{ib}^4} + \frac{1}{2c_{ib}^{(1)}}\tilde{\theta}_{ib}^T\tilde{\theta}_{ib} + \frac{1}{4}z_{ib}^4), \tag{64}$$

where $\tilde{\theta}_{ib} = \theta_{ib}^* - \theta_{ib}, |\eta_{ib}| < \kappa_{ib}(t)$ and $c_{ib}^{(1)}$ is a positive design parameter.

From (13), (51), (63) and (64), one has

$$\begin{aligned} \mathcal{L}V_{bi} = \; & \tfrac{\eta_{ib}^3}{\kappa_{ib}^4-\eta_{ib}^4}[(\eta_{i,b+1} + \tilde{\theta}_{ib}^T\varphi_{ib}(\underline{\hat{\xi}}_{ib}) + z_{i,b+1} + \alpha_{ib} - \tilde{\theta}_{ib}^T\varphi_{ib}(\underline{\hat{\xi}}_{ib}) \\ & + \theta_{ib}^T\varphi_{ib}(\hat{\xi}_{ib}) + g_{ib}(\zeta_i - \hat{\xi}_{i1}) - \dot{\beta}_{ib}] - \tfrac{\eta_{ib}^4\dot{\kappa}_{ib}}{\kappa_{ib}(\kappa_{ib}^4-\eta_{ib}^4)} \\ & + \tfrac{1}{c_{ib}^{(1)}}\tilde{\theta}_{ib}^T\dot{\tilde{\theta}}_{ib} + z_{ib}^3(-\tfrac{z_{ib}}{\varpi_{ib}} + \psi_{ib}) + \tfrac{3}{2}z_{ib}^2 tr(\phi_{ib}^T\phi_{ib}). \end{aligned} \tag{65}$$

By Lemma 2 and $\varphi_{ib}(\underline{\hat{\xi}}_{ib})\varphi_{ib}^T(\underline{\hat{\xi}}_{ib}) \le 1$, it can be derived that

$$-\frac{\eta_{ib}^3}{\kappa_{ib}^4 - \eta_{ib}^4}\tilde{\theta}_{ib}^T\varphi_{ib}(\underline{\hat{\xi}}_{ib}) \leqslant \frac{1}{2}(\frac{\eta_{ib}^3}{\kappa_{ib}^4 - \eta_{ib}^4})^2 + \frac{1}{2}\tilde{\theta}_{ib}^T\tilde{\theta}_{ib}, \tag{66}$$

$$\frac{\eta_{ib}^3}{\kappa_{ib}^4 - \eta_{ib}^4}(\eta_{i,b+1} + z_{i,b+1}) \leqslant \frac{6}{4}(\frac{\eta_{ib}^3}{\kappa_{ib}^4 - \eta_{ib}^4})^{\frac{4}{3}} + \frac{1}{4}z_{i,b+1}^4 + \frac{1}{4}\eta_{i,b+1}^4, \tag{67}$$

$$z_{ib}^3\psi_{ib} \leqslant \frac{3}{4}\bar{\psi}_{ib}^{\frac{4}{3}}\gamma_{ib}^{\frac{4}{3}}z_{ib}^4 + \frac{1}{4\gamma_{ib}^4}, \tag{68}$$

and

$$\frac{3}{2}z_{ib}^2 tr(\phi_{ib}^T\phi_{ib}) \leqslant \frac{3}{4}\imath_{ib}^2 + \frac{3}{4\imath_{ib}^2}\bar{\phi}_{ib}^4 z_{ib}^4, \tag{69}$$

where γ_{ib} and $\imath_{ib}$ are positive constants.

Substituting (66)-(69) into (65), one gets

$$\begin{aligned}\mathcal{L}V_{ib} \leqslant\ & \frac{\eta_{ib}^3}{\kappa_{ib}^4-\eta_{ib}^4}(\alpha_{ib}+\theta_{ib}^T\varphi_{ib}(\hat{\underline{\xi}}_{ib})+g_{ib}(\zeta_i-\hat{\xi}_{i1})-\dot{\beta}_{ib}-\frac{\eta_{ib}\dot{\kappa}_{ib}}{\kappa_{ib}}) \\ & +\frac{1}{4}s_{i,b+1}^4+\frac{1}{4}z_{i,b+1}^4+\frac{1}{2}\left(\frac{\eta_{ib}^3}{\kappa_{ib}^4-\eta_{ib}^4}\right)^2+\frac{1}{2}\tilde{\theta}_{ib}^T\tilde{\theta}_{ib}+\frac{1}{4\gamma_{ib}^4}+\frac{3}{4}\iota_{ib}^2 \\ & +\frac{6}{4}\left(\frac{\eta_{ib}^3}{\kappa_{ib}^4-\eta_{ib}^4}\right)^{\frac{4}{3}}+\frac{1}{c_{ib}^{(1)}}\tilde{\theta}_{ib}^T\left(\frac{\eta_{ib}^3}{\kappa_{ib}^4-\eta_{ib}^4}c_{ib}^{(1)}\varphi_{ib}(\hat{\underline{\xi}}_{ib})-\dot{\theta}_{ib}\right) \\ & +\left(\frac{3}{4}\bar{\psi}_{ib}^{\frac{4}{3}}\gamma_{ib}^{\frac{4}{3}}-\frac{1}{\varpi_{ib}}+\frac{3}{4\iota_{ib}^2}\bar{\phi}_{ib}^4\right)z_{ib}^4.\end{aligned} \tag{70}$$

Choose α_{ib} and $\dot{\theta}_{ib}$ as

$$\begin{aligned}\alpha_{ib} =\ & -c_{ib}^{(3)}\eta_{ib}-g_{ib}(\zeta_i-\hat{\xi}_{i1})-\theta_{ib}^T\varphi_{ib}(\hat{\underline{\xi}}_{ib})-\frac{\eta_{ib}^3}{2(\kappa_{ib}^4-\eta_{ib}^4)} \\ & +\frac{\eta_{ib}\dot{\kappa}_{ib}}{\kappa_{ib}}-\frac{6}{4}\left(\frac{\eta_{ib}^3}{\kappa_{ib}^4-\eta_{ib}^4}\right)^{\frac{1}{3}}-\frac{\eta_{ib}(\kappa_{ib}^4-\eta_{ib}^4)}{4}+\dot{\beta}_{ib},\end{aligned} \tag{71}$$

and

$$\dot{\theta}_{ib}=\frac{\eta_{ib}^3}{\kappa_{ib}^4-\eta_{ib}^4}c_{ib}^{(1)}\varphi_{ib}(\hat{\underline{\xi}}_{ib})-c_{ib}^{(4)}\theta_{ib}, \tag{72}$$

where $c_{ib}^{(3)}>0$ and $c_{ib}^{(4)}>0$ are constants.

Substituting (71)-(72) into (70), it is obvious that

$$\begin{aligned}\mathcal{L}V_{ib} \leqslant\ & -\frac{c_{ib}^{(3)}\eta_{ib}^4}{\kappa_{ib}^4-\eta_{ib}^4}-\frac{1}{4}\eta_{ib}^4+\frac{1}{4}\eta_{i,b+1}^4+\frac{1}{4}z_{i,b+1}^4+\frac{1}{2}\tilde{\theta}_{ib}^T\tilde{\theta}_{ib}+\frac{c_{ib}^{(4)}}{c_{ib}^{(1)}}\tilde{\theta}_{ib}^T\theta_{ib} \\ & +\frac{1}{4\gamma_{ib}^4}+\frac{3}{4}\iota_{ib}^2+\left(\frac{3}{4}\bar{\psi}_{ib}^{\frac{4}{3}}\gamma_{ib}^{\frac{4}{3}}-\frac{1}{\varpi_{ib}}+\frac{3}{4\iota_{ib}^2}\bar{\phi}_{ib}^4\right)z_{ib}^4.\end{aligned} \tag{73}$$

From (64) and (73), we have

$$\begin{aligned}\mathcal{L}V_b =\ & \mathcal{L}[V_{b-1}+\Sigma_{i=1}^N V_{ib}] \\ \leqslant\ & \Sigma_{i=1}^N[-M_i^{(2)}\|\tilde{\xi}_i\|^2-\Sigma_{q=1}^b\frac{c_{iq}^{(3)}s_{iq}^4}{\kappa_{iq}^4-s_{iq}^4}+M_i^{(3)}+\Sigma_{q=1}^b\frac{c_{iq}^{(4)}}{c_{iq}^{(1)}}\tilde{\theta}_{iq}^T\theta_{iq} \\ & +\frac{1}{4}\Sigma_{q=2}^{b+1}z_{iq}^4+\frac{1}{2}\Sigma_{q=2}^b\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}+\Sigma_{j=1}^N a_{ij}\frac{c_{j1}^{(5)}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\theta_{j1}+\Sigma_{q=1}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} \\ & +\frac{1}{4}\eta_{i,b+1}^4+\Sigma_{q=2}^b\frac{1}{4\gamma_{iq}^4}+\Sigma_{q=2}^b\frac{3}{4}\iota_{iq}^2 \\ & +\Sigma_{q=2}^b\left(\frac{3}{4}\bar{\psi}_{iq}^{\frac{4}{3}}\gamma_{iq}^{\frac{4}{3}}-\frac{1}{\varpi_{iq}}+\frac{3}{4\iota_{iq}^2}\bar{\phi}_{iq}^4\right)z_{iq}^4].\end{aligned} \tag{74}$$

Step $\Delta+1$: From (20) (33) and (34), the following result holds

$$\begin{aligned}\dot{\eta}_{i,\Delta+1} =\ & \eta_{i,\Delta+2}+z_{i,\Delta+2}+\alpha_{i,\Delta+1}+\theta_{i,\Delta+1}^T\varphi_{i,\Delta+1}(\hat{\underline{\xi}}_{i,\Delta+1}) \\ & +g_{i,\Delta+1}(\zeta_i-\hat{\xi}_{i1})-\dot{\beta}_{i,\Delta+1}.\end{aligned} \tag{75}$$

Choose the Lyapunov function $V_{\Delta+1}$ as

$$\begin{aligned}V_{\Delta+1} =\ & V_\Delta+\Sigma_{i=1}^N V_{i,\Delta+1} \\ =\ & V_\Delta+\Sigma_{i=1}^N\left(\frac{1}{2}\eta_{i,\Delta+1}^2+\frac{1}{2c_{i,\Delta+1}^{(1)}}\tilde{\theta}_{i,\Delta+1}^T\tilde{\theta}_{i,\Delta+1}+\frac{1}{4}z_{i,\Delta+1}^4\right),\end{aligned} \tag{76}$$

where $\tilde{\theta}_{i,\Delta+1}=\theta_{i,\Delta+1}^*-\theta_{i,\Delta+1}$, and $c_{i,\Delta+1}^{(1)}$ is a positive design parameter.

From (13), (51), (75) and (76), we have that

$$\begin{aligned}\mathcal{L}V_{i,\Delta+1} =\ & \eta_{i,\Delta+1}(\eta_{i,\Delta+2}+z_{i,\Delta+2}+\alpha_{i,\Delta+1}+\tilde{\theta}_{i,\Delta+1}^T\varphi_{i,\Delta+1}(\hat{\underline{\xi}}_{i,\Delta+1}) \\ & -\tilde{\theta}_{i,\Delta+1}^T\varphi_{i,\Delta+1}(\hat{\underline{\xi}}_{i,\Delta+1})+\theta_{i,\Delta+1}^T\varphi_{i,\Delta+1}(\hat{\underline{\xi}}_{i,\Delta+1})-\dot{\beta}_{i,\Delta+1} \\ & +g_{i,\Delta+1}(\zeta_i-\hat{\xi}_{i1}))+\frac{1}{c_{i,\Delta+1}^{(1)}}\tilde{\theta}_{i,\Delta+1}^T\dot{\tilde{\theta}}_{i,\Delta+1} \\ & +z_{i,\Delta+1}^3\left(-\frac{z_{i,\Delta+1}}{\varpi_{i,\Delta+1}}+\psi_{i,\Delta+1}\right)+\frac{3}{2}z_{i,\Delta+1}^2 tr(\phi_{i,\Delta+1}^T\phi_{i,\Delta+1}).\end{aligned} \tag{77}$$

By Lemma 2 and $\varphi_{i,\Delta+1}(\hat{\underline{\xi}}_{i,\Delta+1})\varphi_{i,\Delta+1}^T(\hat{\underline{\xi}}_{i,\Delta+1})\le 1$, one has

$$-\eta_{i,\Delta+1}\tilde{\theta}_{i,\Delta+1}^T\varphi_{i,\Delta+1}(\hat{\underline{\xi}}_{i,\Delta+1})\leqslant\frac{1}{2}\eta_{i,\Delta+1}^2+\frac{1}{2}\tilde{\theta}_{i,\Delta+1}^T\tilde{\theta}_{i,\Delta+1}, \tag{78}$$

$$\eta_{i,\Delta+1}\eta_{i,\Delta+2} \leqslant \frac{1}{2}\eta_{i,\Delta+1}^2 + \frac{1}{2}\eta_{i,\Delta+2}^2, \quad (79)$$

$$\eta_{i,\Delta+1}z_{i,\Delta+2} \leqslant \frac{3}{4}\eta_{i,\Delta+1}^{\frac{4}{3}} + \frac{1}{4}z_{i,\Delta+2}^4, \quad (80)$$

$$z_{i,\Delta+1}^3\psi_{i,\Delta+1} \leqslant \frac{3}{4}\bar{\psi}_{i,\Delta+1}^{\frac{4}{3}}\gamma_{i,\Delta+1}^{\frac{4}{3}}z_{i,\Delta+1}^4 + \frac{1}{4\gamma_{i,\Delta+1}^4}, \quad (81)$$

and

$$\frac{3}{2}z_{i,\Delta+1}^2 tr(\phi_{i,\Delta+1}^T\phi_{i,\Delta+1}) \leqslant \frac{3}{4}\iota_{i,\Delta+1}^2 + \frac{3}{4\iota_{i,\Delta+1}^2}\bar{\phi}_{i,\Delta+1}^4 z_{i,\Delta+1}^4, \quad (82)$$

where $\gamma_{i,\Delta+1}$ and $\iota_{i,\Delta+1}$ are positive constants.

Substituting (78)-(82) into (77), yields that

$$\begin{aligned}\mathcal{L}V_{i,\Delta+1} \leqslant\ & \tfrac{1}{2}\eta_{i,\Delta+1}^2 + \eta_{i,\Delta+1}(\alpha_{i,\Delta+1} + g_{i,\Delta+1}(\zeta_i - \hat{\xi}_{i1}) - \dot{\beta}_{i,\Delta+1} \\ & + \theta_{i,\Delta+1}^T\varphi_{i,\Delta+1}(\hat{\underline{\xi}}_{i,\Delta+1})) + \tfrac{1}{2}\eta_{i,\Delta+2}^2 + \tfrac{1}{2}\tilde{\theta}_{i,\Delta+1}^T\tilde{\theta}_{i,\Delta+1} \\ & + (\tfrac{3}{4}\bar{\psi}_{i,\Delta+1}^{\frac{4}{3}}\gamma_{i,\Delta+1}^{\frac{4}{3}} - \tfrac{1}{\varpi_{i,\Delta+1}} + \tfrac{3}{4\iota_{i,\Delta+1}^2}\bar{\phi}_{i,\Delta+1}^4)z_{i,\Delta+1}^4 \\ & + \tfrac{1}{c_{i,\Delta+1}^{(1)}}\tilde{\theta}_{i,\Delta+1}^T(\eta_{i,\Delta+1}c_{i,\Delta+1}^{(1)}\varphi_{i,\Delta+1}(\hat{\underline{\xi}}_{i,\Delta+1}) - \dot{\theta}_{i,\Delta+1}) \\ & + \tfrac{3}{4}\eta_{i,\Delta+1}^{\frac{4}{3}} + \tfrac{1}{4}z_{i,\Delta+2}^4 + \tfrac{1}{4\gamma_{i,\Delta+1}^4} + \tfrac{3}{4}\iota_{i,\Delta+1}^2.\end{aligned} \quad (83)$$

Choose $\alpha_{i,\Delta+1}$ and $\dot{\theta}_{i,\Delta+1}$ as

$$\begin{aligned}\alpha_{i,\Delta+1} =\ & -c_{i,\Delta+1}^{(3)}\eta_{i,\Delta+1} - g_{i,\Delta+1}(\zeta_i - \hat{\xi}_{i1}) - \tfrac{1}{2}\eta_{i,\Delta+1} - \tfrac{1}{4}\eta_{i,\Delta+1}^3 \\ & - \tfrac{3}{4}\eta_{i,\Delta+1}^{\frac{1}{3}} - \theta_{i,\Delta+1}^T\varphi_{i,\Delta+1}(\hat{\underline{\xi}}_{i,\Delta+1}) + \dot{\beta}_{i,\Delta+1},\end{aligned} \quad (84)$$

and

$$\dot{\theta}_{i,\Delta+1} = \eta_{i,\Delta+1}c_{i,\Delta+1}^{(1)}\varphi_{i,\Delta+1}(\hat{\underline{\xi}}_{i,\Delta+1}) - c_{i,\Delta+1}^{(4)}\theta_{i,\Delta+1}, \quad (85)$$

where $c_{i,\Delta+1}^{(3)} > 0$ and $c_{i,\Delta+1}^{(4)} > 0$ are constants.

Substituting (84) and (85) into (83), it is obvious that

$$\begin{aligned}\mathcal{L}V_{i,\Delta+1} \leqslant\ & -c_{i,\Delta+1}^{(3)}\eta_{i,\Delta+1}^2 - \tfrac{1}{4}\eta_{i,\Delta+1}^4 + \tfrac{1}{2}\eta_{i,\Delta+2}^2 + \tfrac{1}{4}z_{i,\Delta+2}^4 \\ & + \tfrac{1}{4\gamma_{i,\Delta+1}^4} + \tfrac{3}{4}\iota_{i,\Delta+1}^2 + \tfrac{1}{2}\tilde{\theta}_{i,\Delta+1}^T\tilde{\theta}_{i,\Delta+1} + \tfrac{c_{i,\Delta+1}^{(4)}}{c_{i,\Delta+1}^{(1)}}\tilde{\theta}_{i,\Delta+1}^T\theta_{i,\Delta+1} \\ & + (\tfrac{3}{4}\bar{\psi}_{i,\Delta+1}^{\frac{4}{3}}\gamma_{i,\Delta+1}^{\frac{4}{3}} - \tfrac{1}{\varpi_{i,\Delta+1}} + \tfrac{3}{4\iota_{i,\Delta+1}^2}\bar{\phi}_{i,\Delta+1}^4)z_{i,\Delta+1}^4.\end{aligned} \quad (86)$$

From (76) and (86), we have

$$\begin{aligned}\mathcal{L}V_{\Delta+1} =\ & \mathcal{L}[V_\Delta + \Sigma_{i=1}^N V_{i,\Delta+1}] \\ \leqslant\ & \Sigma_{i=1}^N[-M_i^{(2)}\|\tilde{\xi}_i\|^2 - \Sigma_{q=1}^{\Delta}\tfrac{c_{iq}^{(3)}\eta_{iq}^4}{\kappa_{iq}^4 - \eta_{iq}^4} - c_{i,\Delta+1}^{(3)}\eta_{i,\Delta+1}^2 + \tfrac{1}{2}\eta_{i,\Delta+2}^2 \\ & + \Sigma_{q=1}^{\Delta+1}\tfrac{c_{iq}^{(4)}}{c_{iq}^{(1)}}\tilde{\theta}_{iq}^T\theta_{iq} + \tfrac{1}{2}\Sigma_{q=2}^{\Delta+1}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \tfrac{1}{4}\Sigma_{q=2}^{\Delta+2}z_{iq}^4 + \Sigma_{q=1}^{n}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} \\ & + M_i^{(3)} + \Sigma_{j=1}^N a_{ij}\tfrac{c_{j1}^{(5)}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\theta_{j1} + \Sigma_{q=2}^{\Delta+1}\tfrac{1}{4\gamma_{iq}^4} + \Sigma_{q=2}^{\Delta+1}\tfrac{3}{4}\iota_{iq}^2 \\ & + \Sigma_{q=2}^{\Delta+1}(\tfrac{3}{4}\bar{\psi}_{iq}^{\frac{4}{3}}\gamma_{iq}^{\frac{4}{3}} - \tfrac{1}{\varpi_{iq}} + \tfrac{3}{4\iota_{iq}^2}\bar{\phi}_{iq}^4)z_{iq}^4].\end{aligned} \quad (87)$$

Step $s, s = \Delta + 2, \cdots, n-1$: From (20), (33) and (34), one gets

$$\dot{\eta}_{is} = \eta_{i,s+1} + z_{i,s+1} + \alpha_{is} + \theta_{is}^T\varphi_{is}(\hat{\underline{\xi}}_{is}) + g_{is}(\zeta_i - \hat{\xi}_{i1}) - \dot{\beta}_{is}. \quad (88)$$

Choose the Lyapunov function V_s as

$$V_s = V_{s-1} + \Sigma_{i=1}^N V_{is} = V_{s-1} + \Sigma_{i=1}^N(\frac{1}{2}\eta_{is}^2 + \frac{1}{2c_{is}^{(1)}}\tilde{\theta}_{is}^T\tilde{\theta}_{is} + \frac{1}{4}z_{is}^4), \quad (89)$$

where $\tilde{\theta}_{is} = \theta^*_{is} - \theta_{is}$, and $c^{(1)}_{is}$ is a positive design parameter.

From (13), (51), (88) and (89), we can deduce that

$$\begin{aligned}\mathcal{L}V_{is} = \;& \eta_{is}(\eta_{i,s+1} + z_{i,s+1} + \alpha_{is} + \tilde{\theta}^T_{is}\varphi_{is}(\hat{\underline{\xi}}_{is}) + \theta^T_{is}\varphi_{is}(\hat{\underline{\xi}}_{is}) \\ & -\tilde{\theta}^T_{is}\varphi_{is}(\hat{\underline{\xi}}_{is}) - \dot{\beta}_{is} + g_{is}(\zeta_i - \hat{\xi}_{i1})) + \tfrac{1}{c^{(1)}_{is}}\tilde{\theta}^T_{is}\dot{\tilde{\theta}}_{is} \\ & + z^3_{is}(-\tfrac{z_{is}}{\varpi_{is}} + \psi_{is}) + \tfrac{3}{2}z^2_{is}tr(\phi^T_{is}\phi_{is}).\end{aligned} \tag{90}$$

By Lemma 2 and $\varphi_{is}(\hat{\underline{\xi}}_{is})\varphi^T_{is}(\hat{\underline{\xi}}_{is}) \le 1$, one has

$$-\eta_{is}\tilde{\theta}^T_{is}\varphi_{is}(\hat{\underline{\xi}}_{is}) \leqslant \frac{1}{2}\eta^2_{is} + \frac{1}{2}\tilde{\theta}^T_{is}\tilde{\theta}_{is}, \tag{91}$$

$$\eta_{is}\eta_{i,s+1} \leqslant \frac{1}{2}\eta^2_{is} + \frac{1}{2}\eta^2_{i,s+1}, \tag{92}$$

$$\eta_{is}z_{i,s+1} \leqslant \frac{3}{4}\eta^{\frac{4}{3}}_{is} + \frac{1}{4}z^4_{i,s+1}, \tag{93}$$

$$z^3_{is}\psi_{is} \leqslant \frac{3}{4}\bar{\psi}^{\frac{4}{3}}_{is}\gamma^{\frac{4}{3}}_{is}z^4_{is} + \frac{1}{4\gamma^4_{is}}, \tag{94}$$

and

$$\frac{3}{2}z^2_{is}tr(\phi^T_{is}\phi_{is}) \leqslant \frac{3}{4}\iota^2_{is} + \frac{3}{4\iota^2_{is}}\bar{\phi}^4_{is}z^4_{is}, \tag{95}$$

where γ_{is} and ι_{is} are positive constants.

Substituting (91)-(95) into (90), derives that

$$\begin{aligned}\mathcal{L}V_{is} \leqslant \;& \tfrac{1}{2}\eta^2_{is} + \eta_{is}(\alpha_{is} + g_{is}(\zeta_i - \hat{\xi}_{i1}) + \theta^T_{is}\varphi_{is}(\hat{\underline{\xi}}_{is}) - \dot{\beta}_{is}) + \tfrac{1}{2}\eta^2_{i,s+1} \\ & + \tfrac{3}{4}\eta^{\frac{4}{3}}_{is} + \tfrac{1}{4}z^4_{i,s+1} + \tfrac{1}{2}\tilde{\theta}^T_{is}\tilde{\theta}_{is} + (\tfrac{3}{4}\bar{\psi}^{\frac{4}{3}}_{is}\gamma^{\frac{4}{3}}_{is} - \tfrac{1}{\varpi_{is}} + \tfrac{3}{4\iota^2_{is}}\bar{\phi}^4_{is})z^4_{is} \\ & + \tfrac{1}{4\gamma^4_{is}} + \tfrac{3}{4}\iota^2_{is} + \tfrac{1}{c^{(1)}_{is}}\tilde{\theta}^T_{is}(\eta_{is}c^{(1)}_{is}\varphi_{is}(\hat{\underline{\xi}}_{is}) - \dot{\theta}_{is}).\end{aligned} \tag{96}$$

Construct α_{is} and $\dot{\theta}_{is}$ as

$$\alpha_{is} = -c^{(3)}_{is}\eta_{is} - g_{is}(\zeta_i - \hat{\xi}_{i1}) - \theta^T_{is}\varphi_{is}(\hat{\underline{\xi}}_{is}) - \eta_{is} - \frac{3}{4}\eta^{\frac{1}{3}}_{is} + \dot{\beta}_{is}, \tag{97}$$

and

$$\dot{\theta}_{is} = \eta^3_{is}c^{(1)}_{is}\varphi_{is}(\hat{\underline{\xi}}_{is}) - c^{(4)}_{is}\theta_{is}, \tag{98}$$

where $c^{(3)}_{is} > 0$ and $c^{(4)}_{is} > 0$ are constants.

Substituting (97) and (98) into (96), one has

$$\begin{aligned}\mathcal{L}V_{is} \leqslant \;& -c^{(3)}_{iq}\eta^2_{is} - \tfrac{1}{2}\eta^2_{is} + \tfrac{1}{2}\eta^2_{i,s+1} + \tfrac{1}{4}z^4_{i,s+1} + \tfrac{1}{2}\tilde{\theta}^T_{is}\tilde{\theta}_{is} + \tfrac{1}{4\gamma^4_{is}} \\ & + \tfrac{3}{4}\iota^2_{is} + \tfrac{c^{(4)}_{is}}{c^{(2)}_{is}}\tilde{\theta}^T_{is}\theta_{is} + (\tfrac{3}{4}\bar{\psi}^{\frac{4}{3}}_{is}\gamma^{\frac{4}{3}}_{is} - \tfrac{1}{\varpi_{is}} + \tfrac{3}{4\iota^2_{is}}\bar{\phi}^4_{is})z^4_{is}.\end{aligned} \tag{99}$$

From (89) and (99), we have

$$\begin{aligned}\mathcal{L}V_s = \;& \mathcal{L}[V_{s-1} + \Sigma^N_{i=1}V_{is}] \\ \leqslant \;& \Sigma^N_{i=1}[-M^{(2)}_i\|\tilde{\xi}_i\|^2 - \Sigma^{\Delta}_{q=1}\tfrac{c^{(3)}_{iq}\eta^4_{iq}}{\kappa^4_{iq}-\eta^4_{iq}} - \Sigma^s_{q=\Delta+1}c^{(3)}_{iq}\eta^2_{iq} + \tfrac{1}{2}\eta^2_{i,s+1} \\ & + M^{(3)}_i + \Sigma^s_{q=1}\tfrac{c^{(4)}_{iq}}{c^{(1)}_{iq}}\tilde{\theta}^T_{iq}\theta_{iq} + \tfrac{1}{2}\Sigma^s_{q=2}\tilde{\theta}^T_{iq}\tilde{\theta}_{iq} + \tfrac{1}{4}\Sigma^{s+1}_{q=2}z^4_{iq} + \Sigma^n_{q=1}\tilde{\theta}^T_{iq}\tilde{\theta}_{iq} \\ & + \Sigma^N_{j=1}a_{ij}\tfrac{c^{(5)}_{j1}}{c^{(2)}_{j1}}\tilde{\theta}^T_{j1}\theta_{j1} + \Sigma^s_{q=2}\tfrac{1}{4\gamma^4_{iq}} + \Sigma^s_{q=2}\tfrac{3}{4}\iota^2_{iq} \\ & + \Sigma^s_{q=2}(\tfrac{3}{4}\bar{\psi}^{\frac{4}{3}}_{iq}\gamma^{\frac{4}{3}}_{iq} - \tfrac{1}{\varpi_{iq}} + \tfrac{3}{4\iota^2_{iq}}\bar{\phi}^4_{iq})z^4_{iq}].\end{aligned} \tag{100}$$

Step n: An ETC scheme is developed in this step.
•*Control signal*:

$$u_i(t) = h_i(t_k^i), \forall t \in [t_k^i, t_{k+1}^i). \tag{101}$$

•*ETC mechanism*:

$$t_{k+1}^i = \inf\{t > t_k^i \,|\, |\varsigma_i(t)| \geqslant \delta_i |u_i(t)| + m_i\}, \tag{102}$$

where $h_i(t)$ is an intermediate virtual function which will be designed later. $\varsigma_i(t) = h_i(t) - u_i(t), 0 < \delta_i < 1$ and $m_i > 0$ are design parameters. t_k^i is the update time of the controller. When (102) is triggered, the next update time t_{k+1}^i will be generated. $h_i(t_k^i)$ is invariant in $t \in [t_k^i, t_{k+1}^i)$. According to the above ETC rules, $|h_i(t) - u_i(t)| \leqslant \delta_i |u_i(t)| + m_i$ holds in all time.

Remark 6. The number of transmissions generated by the triggering mechanism (102) can be adjusted by design parameters δ_i and m_i. Larger parameter values result in fewer triggers. On the contrary, the number of triggers will increase. When $\delta_i = 0$ and $m_i = 0$, it becomes time-triggered one as the special case of the ETC scheme.

Remark 7. Compared with the time-triggered strategy, the ETC scheme given in (102) allows the control signals to be intermittently sent to the actuator as piecewise constants such that the communication burden from the controller to the actuator can be largely reduced. In addition, the proposed saturation controller can solve the physical limitation problem of the system in actual environment.

Similar to the discussion in [49], the control function $h_i(t)$ satisfies

$$h_i(t) = (1 + \varrho_1(t)\delta_i)u_i(t) + \varrho_2(t)m_i, \tag{103}$$

where $\varrho_1(t)$ and $\varrho_2(t)$ satisfy $|\varrho_1(t)| \le 1$ and $|\varrho_2(t)| \le 1$, respectively.

From (103), one has

$$u_i(t) = \frac{h_i(t)}{1 + \varrho_1(t)\delta_i} - \frac{\varrho_2(t)m_i}{1 + \varrho_1(t)\delta_i}. \tag{104}$$

From (20), (33) and (34), one has

$$\dot{\eta}_{in} = \chi_i(u_i(t))u_i(t) + \theta_{in}^T \varphi_{in}(\hat{\underline{\xi}}_{in}) + g_{in}(\zeta_i - \hat{\xi}_{i1}) - \dot{\beta}_{in}. \tag{105}$$

Choose the Lyapunov function V_n as

$$\begin{aligned} V_n = &\; V_{n-1} + \Sigma_{i=1}^N V_{in} \\ = &\; V_{n-1} + \Sigma_{i=1}^N \left(\tfrac{1}{2}\eta_{in}^2 + \tfrac{1}{2c_{in}^{(1)}}\tilde{\theta}_{in}^T\tilde{\theta}_{in} + \tfrac{l_i^{(1)}}{2l_i^{(2)}}\tilde{\epsilon}_i^2 + \tfrac{1}{4}z_{in}^4\right), \end{aligned} \tag{106}$$

where $\tilde{\theta}_{in} = \theta_{in}^* - \theta_{in}$, and $\epsilon_i = \frac{1}{l_i^{(1)}}, l_i^{(1)}$ is a constant satisfying (6). Let $\hat{\epsilon}_i$ be the estimation of ϵ_i and $\tilde{\epsilon}_i = \epsilon_i - \hat{\epsilon}_i$. $c_{in}^{(1)}$ and $l_i^{(2)}$ are positive design parameters.

From (13), (105) and (106), it can be obtained that

$$\begin{aligned} \mathcal{L}V_{in} = &\; \eta_{in}(\chi_i(u_i(t))u_i(t) + \tilde{\theta}_{in}^T\varphi_{in}(\hat{\underline{\xi}}_{in}) + \theta_{in}^T\varphi_{in}(\hat{\underline{\xi}}_{in}) \\ &-\tilde{\theta}_{in}^T\varphi_{in}(\hat{\underline{\xi}}_{in}) - \dot{\beta}_{in} + g_{in}(\zeta_i - \hat{\xi}_{i1})) + \tfrac{1}{c_{in}^{(1)}}\tilde{\theta}_{in}^T\dot{\tilde{\theta}}_{in} + \tfrac{l_i^{(1)}}{l_i^{(2)}}\tilde{\epsilon}_i\dot{\tilde{\epsilon}}_i \\ &+z_{in}^3\left(-\tfrac{z_{in}}{\varpi_{in}} + \psi_{in}\right) + \tfrac{3}{2}z_{in}^2 tr(\phi_{in}^T\phi_{in}). \end{aligned} \tag{107}$$

By Lemma 2 and $\varphi_{in}(\hat{\underline{\xi}}_{in})\varphi_{in}^T(\hat{\underline{\xi}}_{in}) \le 1$, we have that

$$z_{in}^3\psi_{in} \leqslant \frac{3}{4}\bar{\psi}_{in}^{\frac{4}{3}}\gamma_{in}^{\frac{4}{3}}z_{in}^4 + \frac{1}{4\gamma_{in}^4}, \tag{108}$$

$$-\eta_{in}\tilde{\theta}_{in}^T\varphi_{in}(\hat{\underline{\xi}}_{in}) \leqslant \frac{1}{2}\eta_{in}^2 + \frac{1}{2}\tilde{\theta}_{in}^T\tilde{\theta}_{in}, \tag{109}$$

and

$$\frac{3}{2}z_{in}^2 tr(\phi_{in}^T\phi_{in}) \leqslant \frac{3}{4}\iota_{in}^2 + \frac{3}{4\iota_{in}^2}\bar{\phi}_{in}^4 z_{in}^4, \tag{110}$$

where γ_{in} and ι_{in} are the positive constants.

Substituting (108)-(110) into (107), develops that

$$\begin{aligned}\mathcal{L}V_{in} \leqslant\ & \tfrac{1}{2}\eta_{in}^2 + \eta_{in}(\chi_i(u_i(t))u_i(t) + g_{in}(\zeta_i - \hat{\xi}_{i1}) + \theta_{in}^T\varphi_{in}(\hat{\underline{\xi}}_{in}) \\ & - \dot{\beta}_{in}) + \tfrac{1}{2}\tilde{\theta}_{in}^T\tilde{\theta}_{in} + (\tfrac{3}{4}\bar{\psi}_{in}^{\frac{4}{3}}\gamma_{in}^{\frac{4}{3}} - \tfrac{1}{\varpi_{in}} + \tfrac{3}{4\iota_{in}^2}\bar{\phi}_{in}^4)z_{in}^4 + \tfrac{l_i^{(1)}}{l_i^{(2)}}\tilde{\epsilon}_i\dot{\hat{\epsilon}}_i \\ & + \tfrac{1}{4\gamma_{in}^4} + \tfrac{3}{4}\iota_{in}^2 + \tfrac{1}{c_{in}^{(1)}}\tilde{\theta}_{in}^T(\eta_{in}c_{in}^{(1)}\varphi_{in}(\hat{\underline{\xi}}_{in}) - \dot{\theta}_{in}).\end{aligned} \tag{111}$$

Form (106) and (111), one gets

$$\begin{aligned}\mathcal{L}V_n =\ & \mathcal{L}[V_{n-1} + \Sigma_{i=1}^N V_{in}] \\ \leqslant\ & \Sigma_{i=1}^N[-M_i^{(2)}\|\tilde{\xi}_i\|^2 - \Sigma_{q=1}^{\Delta}\tfrac{c_{iq}^{(3)}\eta_{iq}^4}{\kappa_{iq}^4-\eta_{iq}^4} - \Sigma_{q=\Delta+1}^{n-1}c_{iq}^{(3)}\eta_{iq}^2 + \Sigma_{q=1}^{n-1}\tfrac{c_{iq}^{(4)}}{c_{iq}^{(1)}}\tilde{\theta}_{iq}^T\theta_{iq} \\ & + \tfrac{1}{4}\Sigma_{q=2}^n z_{iq}^4 + \tfrac{1}{2}\Sigma_{q=2}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \Sigma_{j=1}^N a_{ij}\tfrac{c_{j1}^{(5)}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\theta_{j1} + \Sigma_{q=1}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + M_i^{(3)} \\ & + \eta_{in}(\chi_i(u_i(t))u_i(t) + g_{in}(\zeta_i - \hat{\xi}_{i1}) + \theta_{in}^T\varphi_{in}(\hat{\underline{\xi}}_{in}) - \dot{\beta}_{in}) + \eta_{in}^2 \\ & + \Sigma_{q=2}^n\tfrac{1}{4\gamma_{iq}^4} + \Sigma_{q=2}^n\tfrac{3}{4}\iota_{iq}^2 + \tfrac{1}{c_{in}^{(1)}}\tilde{\theta}_{in}^T(\eta_{in}c_{in}^{(1)}\varphi_{in}(\hat{\underline{\xi}}_{in}) - \dot{\theta}_{in}) + \tfrac{l_i^{(1)}}{l_i^{(2)}}\tilde{\epsilon}_i\dot{\hat{\epsilon}}_i \\ & + \Sigma_{q=2}^n(\tfrac{3}{4}\bar{\psi}_{iq}^{\frac{4}{3}}\gamma_{iq}^{\frac{4}{3}} - \tfrac{1}{\varpi_{iq}} + \tfrac{3}{4\iota_{iq}^2}\bar{\phi}_{iq}^4)z_{iq}^4].\end{aligned} \tag{112}$$

From (104), the intermediate continuous control function $h_i(t)$ is designed as

$$h_i(t) = -(1+\delta_i)(\alpha_{in}\hat{\epsilon}_i\tanh(\frac{\eta_{in}\alpha_{in}\hat{\epsilon}_i}{\vartheta_i}) + \frac{m_i}{1-\delta_i}\tanh(\frac{\eta_{in}m_i}{(1-\delta_i)\vartheta_i})). \tag{113}$$

From Lemma 3 and (104), one has

$$\begin{aligned}\tfrac{\eta_{in}\chi_i(u_i(t))h_i(t)}{1+\varrho_1(t)\delta_i} \leqslant\ & -\eta_{in}\alpha_{in} + l_i^{(1)}\tilde{\epsilon}_i|\eta_{in}\alpha_{in}| + 0.2785\vartheta_i \\ & - \tfrac{\eta_{in}\chi_i(u_i(t))m_i}{1-\delta_i}\tanh(\tfrac{\eta_{in}m_i}{(1-\delta_i)\vartheta_i}),\end{aligned} \tag{114}$$

and

$$-\frac{\eta_{in}\chi_i(u_i(t))\varrho_2(t)m_i}{1+\varrho_1(t)\delta_i} \leqslant \frac{|\eta_{in}|\chi_i(u_i(t))|\varrho_2(t)|m_i}{1+\varrho_1(t)\delta_i} \leqslant \frac{|\eta_{in}|\chi_i(u_i(t))m_i}{1-\delta_i}. \tag{115}$$

From (104), (114), (115) and Lemma 3, we have

$$\eta_{in}\chi_i(u_i(t))u_i(t) \leqslant -\eta_{in}\alpha_{in} + l_i^{(1)}\tilde{\epsilon}_i|\eta_{in}\alpha_{in}| + 0.557\vartheta_i. \tag{116}$$

By substituting (116) into (112), the following holds that

$$\begin{aligned}\mathcal{L}V_n =\ & \mathcal{L}[V_{n-1} + \Sigma_{i=1}^N V_{in}] \\ \leqslant\ & \Sigma_{i=1}^N[-M_i^{(2)}\|\tilde{\xi}_i\|^2 - \Sigma_{q=1}^{\Delta}\tfrac{c_{iq}^{(3)}\eta_{iq}^4}{\kappa_{iq}^4-\eta_{iq}^4} - \Sigma_{q=\Delta+1}^{n-1}c_{iq}^{(3)}\eta_{iq}^2 + \Sigma_{q=1}^{n-1}\tfrac{c_{iq}^{(4)}}{c_{iq}^{(1)}}\tilde{\theta}_{iq}^T\theta_{iq} \\ & + \tfrac{1}{4}\Sigma_{q=2}^n z_{iq}^4 + \tfrac{1}{2}\Sigma_{q=2}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + \Sigma_{j=1}^N a_{ij}\tfrac{c_{j1}^{(5)}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\theta_{j1} + \Sigma_{q=1}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq} + M_i^{(3)} \\ & + \eta_{in}(-\alpha_{in} + g_{in}(\zeta_i - \hat{\xi}_{i1}) + \theta_{in}^T\varphi_{in}(\hat{\underline{\xi}}_{in}) - \dot{\beta}_{in}) + \eta_{in}^2 + 0.557\vartheta_i \\ & + \Sigma_{q=2}^n\tfrac{1}{4\gamma_{iq}^4} + \Sigma_{q=2}^n\tfrac{3}{4}\iota_{iq}^2 + \tfrac{1}{c_{in}^{(1)}}\tilde{\theta}_{in}^T(\eta_{in}c_{in}^{(1)}\varphi_{in}(\hat{\underline{\xi}}_{in}) - \dot{\theta}_{in}) \\ & + \tfrac{l_i^{(1)}}{l_i^{(2)}}\tilde{\epsilon}_i(l_i^{(2)}|\eta_{in}\alpha_{in}| + \dot{\hat{\epsilon}}_i) + \Sigma_{q=2}^n(\tfrac{3}{4}\bar{\psi}_{iq}^{\frac{4}{3}}\gamma_{iq}^{\frac{4}{3}} - \tfrac{1}{\varpi_{iq}} + \tfrac{3}{4\iota_{iq}^2}\bar{\phi}_{iq}^4)z_{iq}^4].\end{aligned} \tag{117}$$

By (117), design adaptive laws $\dot{\theta}_{in}, \dot{\hat{\epsilon}}_i$ and α_{in} as

$$\alpha_{in} = c_{in}^{(3)}\eta_{in} + g_{in}(\zeta_i - \hat{\xi}_{i1}) + \theta_{in}^T\varphi_{in}(\hat{\underline{\xi}}_{in}) + \eta_{in} - \dot{\beta}_{in}, \tag{118}$$

$$\dot{\theta}_{in} = \eta_{in}c_{in}^{(1)}\varphi_{in}(\hat{\underline{\xi}}_{in}) - c_{in}^{(4)}\theta_{in}, \tag{119}$$

and

$$\dot{\hat{\epsilon}}_i = l_i^{(2)}|\eta_{in}\alpha_{in}| - l_i^{(3)}\hat{\epsilon}_i. \tag{120}$$

Substituting (118)-(120) into (117), one gets

$$\begin{aligned}\mathcal{L}V_n = & \ \mathcal{L}[V_{n-1}+\Sigma_{i=1}^N V_{in}] \\ \leqslant & \ \Sigma_{i=1}^N[-M_i^{(2)}\|\tilde{\xi}_i\|^2-\Sigma_{q=1}^{\Delta}\frac{c_{iq}^{(3)}\eta_{iq}^4}{\kappa_{iq}^4-\eta_{iq}^4}-\Sigma_{q=\Delta+1}^n c_{iq}^{(3)}\eta_{iq}^2+\Sigma_{q=1}^n\frac{c_{iq}^{(4)}}{c_{iq}^{(1)}}\tilde{\theta}_{iq}^T\theta_{iq} \\ & +\tfrac{1}{4}\Sigma_{q=2}^n z_{iq}^4+0.557\vartheta_i+\tfrac{1}{2}\Sigma_{q=2}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}+\Sigma_{j=1}^N a_{ij}\frac{c_{j1}^{(5)}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\theta_{j1} \\ & +\Sigma_{q=1}^n\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}+\Sigma_{q=2}^n\frac{1}{4\gamma_{iq}^4}+M_i^{(3)}+\Sigma_{q=2}^n\tfrac{3}{4}\iota_{iq}^2+\frac{l_i^{(1)}l_i^{(3)}}{l_i^{(2)}}\tilde{\epsilon}_i\hat{\epsilon}_i \\ & +\Sigma_{q=2}^n(\tfrac{3}{4}\bar{\psi}_{iq}^{\frac{4}{3}}\gamma_{iq}^{\frac{4}{3}}-\frac{1}{\varpi_{iq}}+\frac{3}{4\iota_{iq}^2}\bar{\phi}_{iq}^4)z_{iq}^4].\end{aligned} \tag{121}$$

By completing the square, we have that

$$\frac{c_{iq}^{(4)}}{c_{iq}^{(1)}}\tilde{\theta}_{iq}^T\theta_{iq}\leqslant-\frac{c_{iq}^{(4)}}{2c_{iq}^{(1)}}\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}+\frac{c_{iq}^{(4)}}{2c_{iq}^{(1)}}\tilde{\theta}_{iq}^{*T}\tilde{\theta}_{iq}^*, \tag{122}$$

$$\frac{c_{j1}^{(5)}}{c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\theta_{j1}\leqslant-\frac{c_{j1}^{(5)}}{2c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\tilde{\theta}_{j1}+\frac{c_{j1}^{(5)}}{2c_{j1}^{(2)}}\tilde{\theta}_{j1}^{*T}\tilde{\theta}_{j1}^*, \tag{123}$$

and

$$\frac{l_i^{(1)}l_i^{(3)}}{l_i^{(2)}}\tilde{\epsilon}_i\hat{\epsilon}_i\leqslant-\frac{l_i^{(1)}l_i^{(3)}}{2l_i^{(2)}}\tilde{\epsilon}_i^2+\frac{l_i^{(1)}l_i^{(3)}}{2l_i^{(2)}}\epsilon_i^2=-\frac{l_i^{(1)}l_i^{(3)}}{2l_i^{(2)}}\tilde{\epsilon}_i^2+\frac{l_i^{(3)}}{2l_i^{(1)}l_i^{(2)}}, \tag{124}$$

Taking (122)-(124) into (121) and from Lemma 5, one gets

$$\begin{aligned}\mathcal{L}V_n \leqslant & \ \Sigma_{i=1}^N[-M_i^{(2)}\|\tilde{\xi}_i\|^2-\Sigma_{q=1}^{\Delta}c_{iq}^{(3)}\log\frac{\kappa_{iq}^4}{\kappa_{iq}^4-\eta_{iq}^4}-\Sigma_{q=\Delta+1}^n c_{iq}^{(3)}\eta_{iq}^4 \\ & -(\frac{c_{i1}^{(4)}}{2c_{i1}^{(1)}}-1)\tilde{\theta}_{i1}^T\tilde{\theta}_{i1}-\Sigma_{j=1}^N a_{ij}\frac{c_{j1}^{(5)}}{2c_{j1}^{(2)}}\tilde{\theta}_{j1}^T\tilde{\theta}_{j1}-\frac{l_i^{(1)}l_i^{(3)}}{2l_i^{(2)}}\tilde{\epsilon}_i^2 \\ & -\Sigma_{q=2}^n(\frac{c_{iq}^{(4)}}{2c_{iq}^{(1)}}-\tfrac{3}{2})\tilde{\theta}_{iq}^T\tilde{\theta}_{iq}-\Sigma_{q=2}^n(\frac{1}{\varpi_{iq}}-\tfrac{3}{4}\bar{\psi}_{iq}^{\frac{4}{3}}\gamma_{iq}^{\frac{4}{3}}-\frac{3}{4\iota_{iq}^2}\bar{\phi}_{iq}^4-\tfrac{1}{4})z_{iq}^4 \\ & +\Sigma_{j=1}^N a_{ij}\frac{c_{j1}^{(5)}}{2c_{j1}^{(2)}}\theta_{j1}^{*T}\theta_{j1}^*+\Sigma_{q=1}^n\frac{c_{iq}^{(4)}}{2c_{iq}^{(1)}}\theta_{iq}^{*T}\theta_{iq}^*+\frac{l_i^{(3)}}{2l_i^{(1)}l_i^{(2)}}+0.557\vartheta_i \\ & +M_i^{(3)}+\Sigma_{q=2}^n\frac{1}{4\gamma_{iq}^4}+\Sigma_{q=2}^n\tfrac{3}{4}\iota_{iq}^2].\end{aligned} \tag{125}$$

Select parameters $c_{i1}^{(4)}, c_{i1}^{(1)}, c_{iq}^{(1)}, c_{iq}^{(4)}, \varpi_{iq}, \bar{\psi}_{iq}, \gamma_{iq}, \iota_{iq}, \bar{\phi}_{iq}, q=2,\cdots,n,$ such that $c_{i1}^{(4)}-2c_{i1}^{(1)}>0, c_{iq}^{(4)}-3c_{iq}^{(1)}>0$ and $\frac{4}{\varpi_{iq}}-3\bar{\psi}_{iq}^{\frac{4}{3}}\gamma_{iq}^{\frac{4}{3}}-\frac{3}{\iota_{iq}^2}\bar{\phi}_{iq}^4-1>0$. Let

$$\begin{aligned}\flat = & \ \min\{\frac{M_i^{(2)}}{\lambda_{max}(P)}, c_{i1}^{(3)}, c_{iq}^{(3)}, l_i^{(3)}, c_{i1}^{(4)}-2c_{i1}^{(1)}, \\ & c_{iq}^{(4)}-3c_{iq}^{(1)}, \frac{4}{\varpi_{iq}}-3\bar{\psi}_{iq}^{\frac{4}{3}}\gamma_{iq}^{\frac{4}{3}}-\frac{3}{\iota_{iq}^2}\bar{\phi}_{iq}^4-1\},\end{aligned} \tag{126}$$

for $i=1,\cdots,N, q=2,\cdots,n,$

$$\begin{aligned}M_i^{(4)} = & \ M_i^{(3)}+\Sigma_{j=1}^N a_{ij}\frac{c_{j1}^{(5)}}{2c_{j1}^{(2)}}\theta_{j1}^{*T}\theta_{j1}^*+\Sigma_{q=1}^n\frac{c_{iq}^{(4)}}{2c_{iq}^{(1)}}\theta_{iq}^{*T}\theta_{iq}^*+\frac{l_i^{(3)}}{2l_i^{(1)}l_i^{(2)}} \\ & +0.557\vartheta_i+\Sigma_{q=2}^n\frac{1}{4\gamma_{iq}^4}+\Sigma_{q=2}^n\tfrac{3}{4}\iota_{iq}^2,\end{aligned} \tag{127}$$

and

$$M^{(4)}=\Sigma_{i=1}^N M_i^{(4)}. \tag{128}$$

The following inequality holds

$$\mathcal{L}V_n\leqslant-\flat V_n+M^{(4)}, \tag{129}$$

5. Analysis of Stability

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Theorem 1. Consider the nonlinear SMAS (2), under Assumptions 2–5, the state observer (20), and the ETC scheme (101), (102),(113), associated with the adaptive laws (46), (47), (60), (72), (85), (98), (119) and (120), and the intermediate control functions (45), (59), (71), (84), (97) and (118), the consensus tracking can be achieved with consensus errors remaining within small neighborhoods of the origin. Moreover, the following objectives can be guaranteed:

(i) The error signals η_{iq}, the observer errors $\tilde{\xi}_i$, the adaptive parameter errors $\tilde{\theta}_{iq}$ and $\tilde{\epsilon}_i$ satisfy the following bound conditions:

$$E(|\eta_{iq}|) \leqslant \kappa_{iq}(t)(1-e^{-4V_n(0)-\frac{4M^{(4)}}{\flat}})^{\frac{1}{4}}, q=1,\cdots,\Delta, \tag{130}$$

$$E(|\eta_{iq}|) \leqslant (2V_n(0)+\frac{2M^{(4)}}{\flat})^{\frac{1}{2}}, q=\Delta+1,\cdots,n, \tag{131}$$

$$E(\|\tilde{\xi}_i\|) \leqslant (\frac{1}{\lambda_{min}(P_i)})^{\frac{1}{2}}(V_n(0)+\frac{M^{(4)}}{\flat})^{\frac{1}{2}}, \tag{132}$$

$$E(|\tilde{\theta}_{iq}|) \leqslant (2\gamma_{iq}V_n(0)+\frac{2\gamma_{iq}M^{(4)}}{\flat})^{\frac{1}{2}}, q=1,\cdots,n, \tag{133}$$

and

$$E(|\tilde{\epsilon}_i|) \leqslant (\frac{2\bar{\gamma}_{iq}}{\pi}(V_n(0)+\frac{M^{(4)}}{\flat}))^{\frac{1}{2}}, \tag{134}$$

for $i=1,\cdots,N$.

(ii) System output and partial state constraints are ensured, ie., $\xi_{iq} < k_{c_{iq}}(t), q=1,2,\cdots,\Delta, \forall t>0$.

(iii) All system signals are bounded in probability.

(iv) The Zeno behavior can be avoided.

Proof: From Lemma 4 and the fact of $0<e^{-\flat t}<1$, one has

$$E[V(t)] \leqslant V_n(0)e^{-\flat t}+\frac{M^{(4)}}{\flat} \leqslant V_n(0)+\frac{M^{(4)}}{\flat}, \tag{135}$$

and then

$$E[V(t)] \leqslant \frac{M^{(4)}}{\flat}, t\to\infty. \tag{136}$$

From (106) and (135), the following inequality can be obtained

$$E(\frac{1}{4}\log\frac{\kappa_{iq}^4}{\kappa_{iq}^4-\eta_{iq}^4}) \leqslant V_n(0)+\frac{M^{(4)}}{\flat}, q=1,\cdots,\Delta, \tag{137}$$

Taking the exponent on both sides of (137), yields that

$$E(|\eta_{iq}|) \leqslant \kappa_{iq}(t)(1-e^{-4V_n(0)-\frac{4M^{(4)}}{\flat}})^{\frac{1}{4}} < \kappa_{iq}(t), q=1,\cdots,\Delta.$$

Let $\eta_1=[\eta_{11},\eta_{21},\cdots,\eta_{N1}]^T, \zeta=[\zeta_1,\zeta_2,\cdots,\zeta_N]^T, \bar{\zeta}=\zeta-1_N\zeta_0$ and $\bar{\kappa}_1(t)=\max_{i=1,\cdots,N}\{\kappa_{i1}(t)\}$. Since $\eta_1=\mathcal{L}_1(\zeta-1_N\zeta_0)$, thus, from Lemma 1, $\mathcal{L}_1$ is invertible. Then $|\zeta_i-\zeta_0| \leqslant \|\bar{\zeta}\|_\infty \leqslant \|\mathcal{L}_1\|_\infty\|\eta_1\|_\infty \leqslant \|\mathcal{L}_1\|_\infty\bar{\kappa}_1(t)(1-e^{-4V_n(0)-\frac{4M^{(4)}}{\flat}})^{\frac{1}{4}}$. Therefore, the consensus errors remaining within small neighborhoods of the origin are ensured.

(i) From (106) and (135), one has

$$E(\frac{1}{2}\eta_{iq}^2) \leqslant V_n(0)+\frac{M^{(4)}}{\flat}, q=\Delta+1,\cdots,n, \tag{138}$$

and

$$E(\tilde{\xi}_i^T P_i\tilde{\xi}_i) \leqslant V_n(0)+\frac{M^{(4)}}{\flat}. \tag{139}$$

Thus, (131) and (132) can be obtained. Similarly, we can obtain (133) and (134).

(ii) According to Assumption 2, one has $|\zeta_0| \leqslant a_0$. Thus $|\xi_{i1}| \leqslant \|\zeta\|_\infty \leqslant \|\mathcal{L}_1^{-1}\|_\infty \|\eta_1\|_\infty + a_0 \leqslant \|\mathcal{L}_1^{-1}\|_\infty \max_{i=1,\cdots,N}\{\kappa_{i1}(t)\} + a_0$. Choosing $\kappa_{i1}(t) \leqslant \frac{k_{c_{i1}}(t) - a_0}{\|\mathcal{L}_1^{-1}\|_\infty}$, we can obtain that $|\xi_{i1}| < k_{c_{i1}}(t)$.
According to $\eta_{i2} = \hat{\xi}_{i2} - \beta_{i2} = \xi_{i2} - \tilde{\xi}_{i2} - z_{i2} - \alpha_{i1}$, one has $|\xi_{i2}| \leqslant |\tilde{\xi}_{i2}| + |\alpha_{i1}| + |\eta_{i2}| + |z_{i2}|$. Because α_{i1} is continuous, there exists a constant $\bar{a}_{i1}$ such that $|\alpha_{i1}| \leqslant \bar{a}_{i1}$. From (106) and (135), one has $E(|z_{iq}|) \leqslant (4V_n(0) + \frac{4M^{(4)}}{\flat})^{\frac{1}{4}}$. Letting

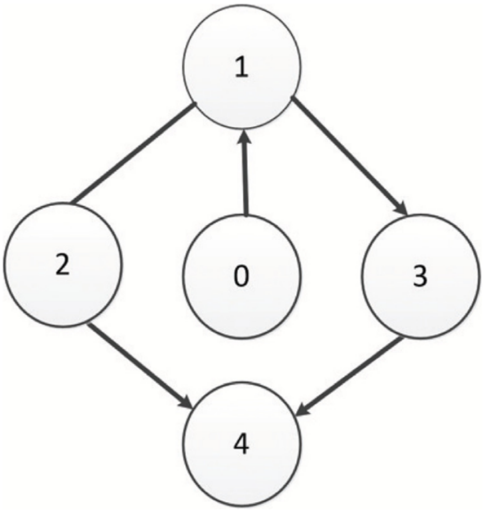

Fig. 1. Connected graph.

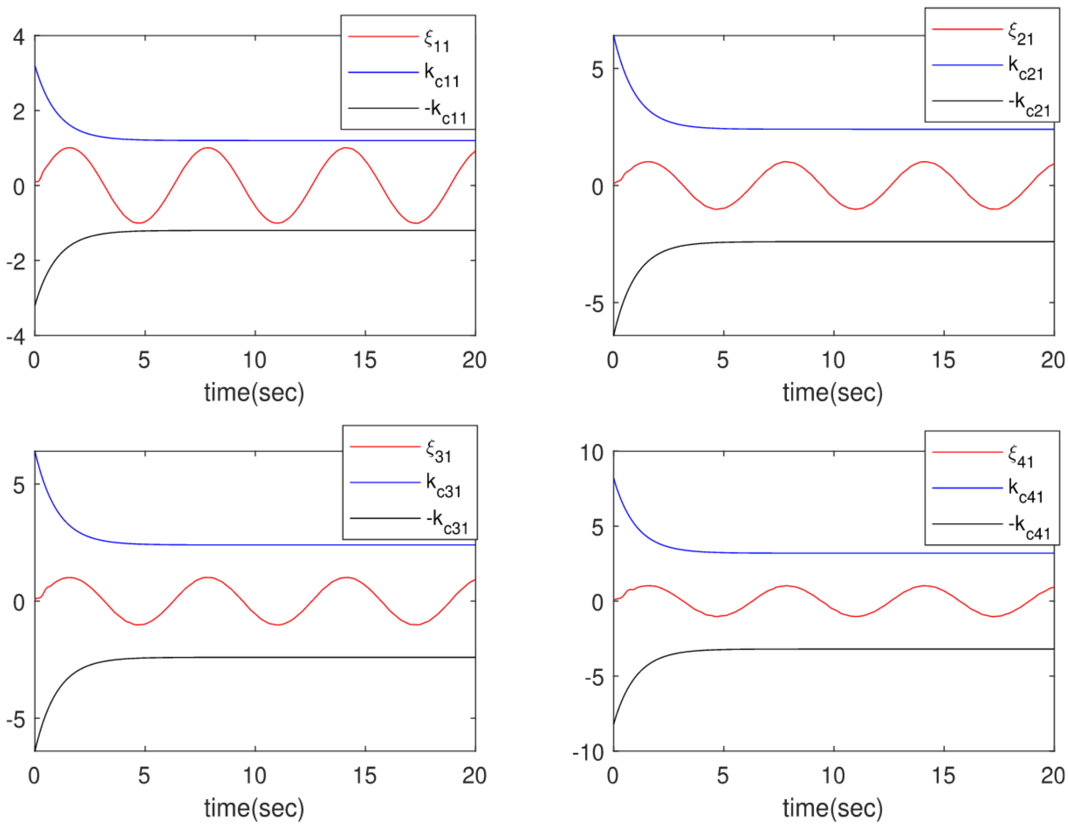

Fig. 2. The constrains on state $\xi_{11}, \xi_{21}, \xi_{31}$ and ξ_{41}.

$\bar{c}_i = \sqrt{(V_n(0) + \frac{M^{(4)}}{\flat})/\lambda_{min}(P_i)}, \bar{r}_i = (4V_n(0) + \frac{4M^{(4)}}{\flat})^{\frac{1}{4}}$ and choosing $\kappa_{i2}(t) = k_{c_{i2}}(t) - \bar{c}_i - \bar{a}_{i1} - \bar{r}_i$, we can obtain $|\xi_{i2}| < k_{c_{i2}}(t)$. Similar to ξ_{i2} and choosing $\kappa_{iq}(t) = k_{c_{iq}}(t) - \bar{c}_i - \bar{a}_{i,q-1} - \bar{r}_i$, we can obtain $|\xi_{iq}| < k_{c_{iq}}(t)$ for $q = 3, \cdots, \Delta$. Therefore, the time-varying constraints for partial states are never violated.

(iii) Since $\eta_{i,\Delta+1} = \hat{\xi}_{i,\Delta+1} - \beta_{i,\Delta+1} = \xi_{i,\Delta+1} - \tilde{\xi}_{i,\Delta+1} - z_{i,\Delta+1} - \alpha_{i\Delta}$, thus $|\xi_{i,\Delta+1}| \leqslant |\tilde{\xi}_{i,\Delta+1}| + |\alpha_{i\Delta}| + |\eta_{i,\Delta+1}| + |z_{i,\Delta+1}|$.From the boundedness of $\tilde{\xi}_{i,\Delta+1}, \alpha_{i\Delta}, \eta_{i,\Delta+1}$ and $z_{i,\Delta+1}$, we can obtain that $\xi_{i,\Delta+1}$ are bounded.Similar to $\xi_{i,\Delta+1}, \xi_{iq}, q = \Delta+2, \cdots, n$, are bounded.Therefore, the unconstrained states are also bounded.Since $\tilde{\xi}_i$ and ξ_i are bounded and $\tilde{\xi}_i = \xi_i - \hat{\xi}_i$, thus, $\hat{\xi}_i$ is bounded.In addition, from (133) and(134), one has $|\hat{\epsilon}_i| \leqslant \epsilon_i + |\tilde{\epsilon}_i| \leqslant \epsilon_i + (\frac{2\bar{\gamma}_i}{l_i^{(1)}}(V_n(0) + \frac{M^{(4)}}{v}))^{\frac{1}{2}}$ and $|\hat{\theta}_{iq}| \leqslant |\theta_{iq}| + |\tilde{\theta}_{iq}| \leqslant |\theta_{iq}| + (2\gamma_{iq}V_n(0) + \frac{2\gamma_{iq}M^{(4)}}{v})^{\frac{1}{2}}, q = 1, \cdots, n$, respectively. Then, all systems signals are bounded.

(iv) Let us prove that no Zeno behavior happens. It needs to prove that $t_{k+1}^i - t_k^i \geqslant t^* > 0$ for all i and k. Since $\varsigma_i(t) = h_i(t) - u_i(t), \forall t \in [t_k^i, t_{k+1}^i)$, thus

$$\frac{d}{dt}|\varsigma_i| = \frac{d}{dt}(\varsigma_i^2)^{\frac{1}{2}} = \text{sgn}(\varsigma_i)\dot{\varsigma}_i \leqslant |\dot{h}_i|. \quad (140)$$

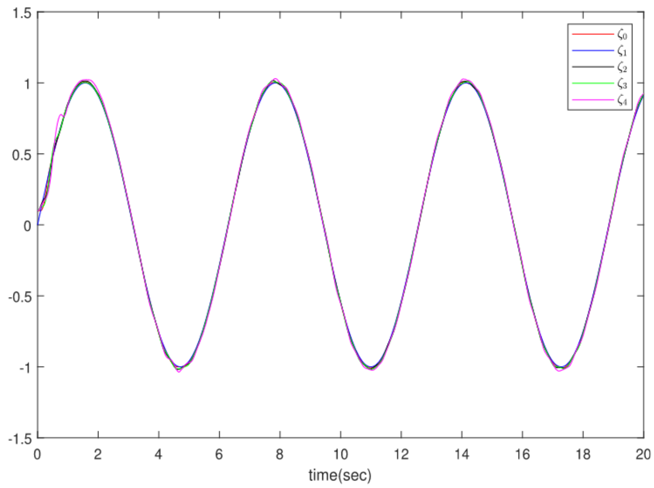

Fig. 3. The trajectories of $\zeta_0, \zeta_1, \zeta_2, \zeta_3$, and ζ_4 with ETC method.

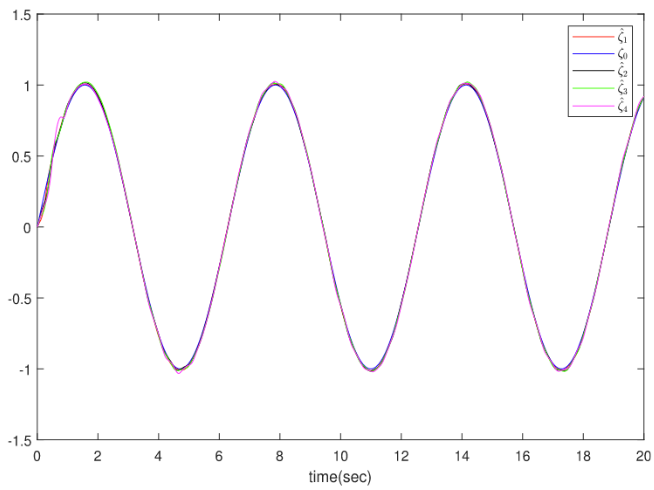

Fig. 4. The trajectories of $\zeta_0, \hat{\zeta}_1, \hat{\zeta}_2, \hat{\zeta}_3$, and $\hat{\zeta}_4$ with ETC method.

From (113) and (118), $\dot{h}_i$ is a function of bounded signals. Thus, $|\dot{h}_i| \leqslant c$ with $c > 0$ being a constant. Note that $\lim_{t \to t^i_{k+1}} \varsigma_i = \delta_i |u_i(t)| + m_i$ and $\varsigma_i(t^i_k) = 0$. Integrating both sides of (140) from t^i_k to t^i_{k+1} and letting $t^* = \frac{\delta_i |u_i(t)| + m_i}{c} > 0$, the lower bound t^* of inter-execution interval is obtained such that $t^i_{k+1} - t^i_k \geqslant t^* > 0$. Therefore, the Zeno behavior does not occur. □.

Remark 8. By selecting appropriate parameters, stability of closed-loop system is ensured by (129). From (129), we know that larger $\flat$ and smaller $M^{(4)}$ lead to faster convergence and smaller error bounds. However, there is a contradiction between fast convergence and smaller error bounds. This requires us to find a balance between the fast convergence and an appropriate error bound by the selection of appropriate parameters.

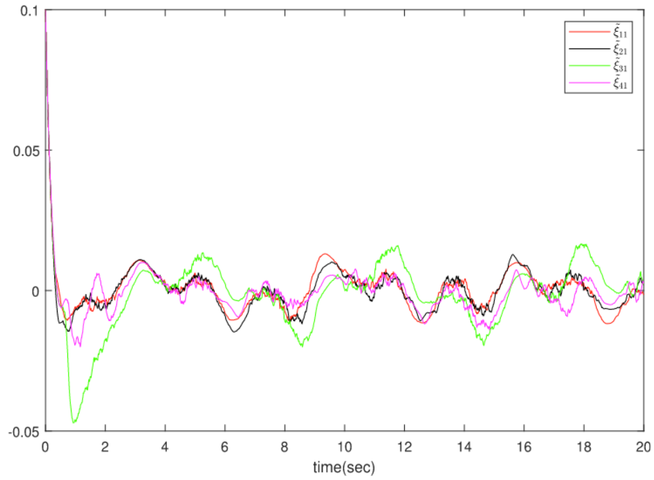

Fig. 5. The trajectories of $\tilde{\xi}_{11}, \tilde{\xi}_{21}, \tilde{\xi}_{31}$, and $\tilde{\xi}_{41}$ with ETC method.

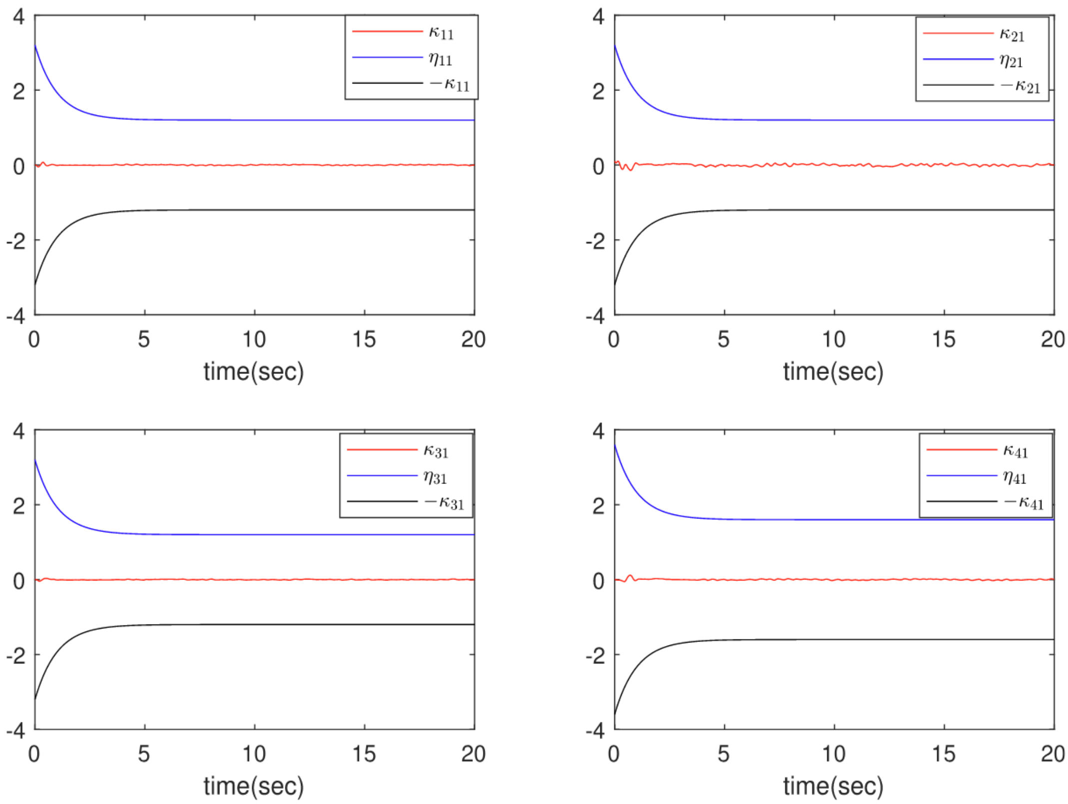

Fig. 6. The constrains on error variable $\eta_{11}, \eta_{21}, \eta_{31}$ and η_{41}.

6. Simulation

An example is provided in this section to illustrate the validity of the proposed ETC scheme.

Example: Consider a SMAS consisting of four followers and a leader as the reference signal. The communication relationship among four followers and the leader can be modeled by a directed graph containing a spanning tree shown in Fig. 1. Each follower is described as

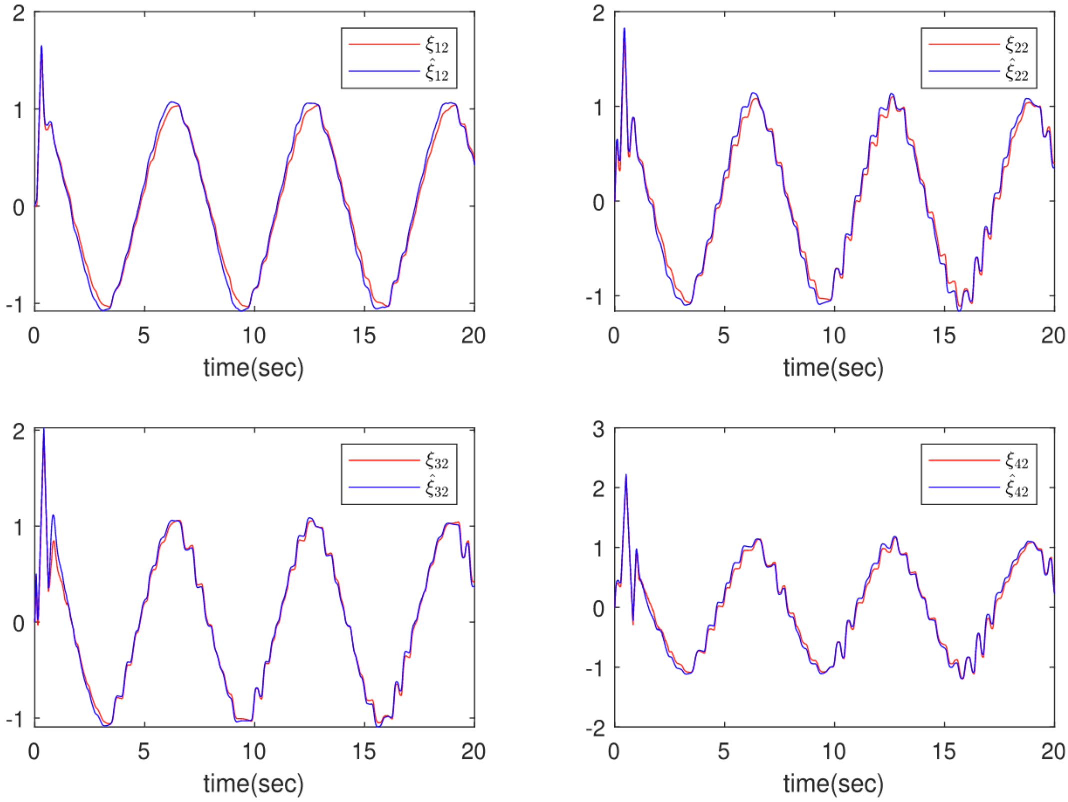

Fig. 7. The trajectories of $\xi_{12}, \hat{\xi}_{12}, \xi_{22}, \hat{\xi}_{22}, \xi_{32}, \hat{\xi}_{32}, \xi_{42}$, and $\hat{\xi}_{42}$ with ETC method.

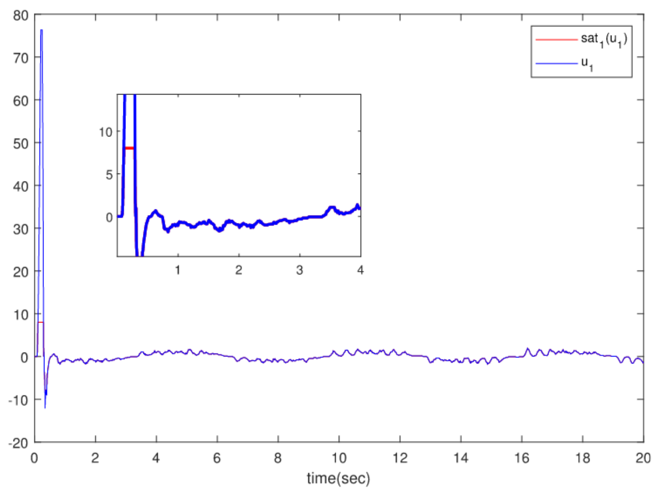

Fig. 8. Input $u_1(t)$ and saturated input $\mathrm{sat}_1(u_1(t))$.

$$\begin{cases} d\xi_{i1} &= (\xi_{i2} + d_{i1}(\xi_i, t) + f_{i1}(\xi_{i1}))dt + p_{i1}(\xi_i)d\omega, \\ d\xi_{i2} &= (\mathrm{sat}_i(u_i(t)) + d_{i2}(\xi_i, t) + f_{i2}(\xi_i))dt + p_{i2}(\xi_i)d\omega, \\ \zeta_i &= \xi_{i1}, i = 1, 2, 3, 4, \end{cases} \quad (141)$$

where $\mathrm{sat}_i(u_i(t))$ is given in (4), $d_{i1}(\xi_i, t) = 0.1\sin(\xi_{i1}^2\xi_{i2}), d_{i2}(\xi_i, t) = 0.1\sin(\xi_{i1}\xi_{i2}^2), f_{i1}(\xi_{i1}) = -0.1\sin(\xi_{i1}), f_{i2}(\xi_i) = 0.1\xi_{i1}\cos(\xi_{i2})$, $p_{i1}(\xi_i) = 0.5\sin(\xi_{i1})\cos(\xi_{i2})$ and $p_{i2}(\xi_i) = 0.5\sin(2\xi_{i1}\xi_{i2}^2)$. Only state ξ_{i1} are required to be within the specified area. The given reference signal is $\zeta_0 = \sin(t)$. Choosing $g_{i1} = 5$ and $g_{i2} = 10$, the state observer (20) can be written as

$$\begin{cases} \dot{\hat{\xi}}_{i1} = & \hat{\xi}_{i2} + \theta_{i1}^T\varphi_{i1}(\hat{\xi}_{i1}) + 5(y_i - \hat{\xi}_{i1}), \\ \dot{\hat{\xi}}_{i2} = & \chi_i(u_i(t))u_i(t) + \theta_{i2}^T\varphi_{i2}(\hat{\underline{\xi}}_i) + 10(y_i - \hat{\xi}_{i1}), \\ \hat{\zeta}_i = & \hat{\xi}_{i1}, i = 1, 2, 3, 4, \end{cases} \quad (142)$$

Set the design parameters of $u_i(t)$ in (101), the trigger condition in (102), $h_i(t)$ in (113), the virtual controller α_{i1} in (45) and α_{i2} in (118), the adaptive laws $\dot{\theta}_{i1}$ in (46), $\dot{\theta}_{i2}$ in (119) and $\dot{\hat{\epsilon}}_i$ in (120), the saturated control $\mathrm{sat}_i(u_i(t))$ in (3) as: $c_{11}^{(3)} = c_{21}^{(3)} = 15, c_{31}^{(3)} = 13, c_{41}^{(3)} = 12, c_{12}^{(3)} = c_{22}^{(3)} = c_{32}^{(3)} = 70, c_{42}^{(3)} = 65, k_{c_{11}} = k_{c_{21}} = 2e^{-t} + 1.2, k_{c_{31}} = 4e^{-t} + 2.4, k_{c_{41}} = 4e^{-t} + 2.6$, $\varpi_{i2} = 0.05, \delta_i = 0.5, u_{im} = 8, m_i = 0.1, \vartheta_1 = \vartheta_2 = 1, \vartheta_3 = \vartheta_4 = 0.5, c_{11}^{(1)} = c_{21}^{(1)} = c_{12}^{(1)} = c_{22}^{(1)} = 50, c_{31}^{(1)} = c_{41}^{(1)} = c_{32}^{(1)} = c_{42}^{(1)} = 20$, $c_{11}^{(4)} = c_{21}^{(4)} = c_{12}^{(4)} = c_{22}^{(4)} = 25, c_{31}^{(4)} = c_{41}^{(4)} = c_{32}^{(4)} = c_{42}^{(4)} = 4, l_i^{(2)} = 0.5, l_i^{(3)} = 1$, for $1 < i < 4$. All the initial states are set to 0 apart from $\xi_{11} = \xi_{21} = \xi_{31} = \xi_{41} = 0.1$.

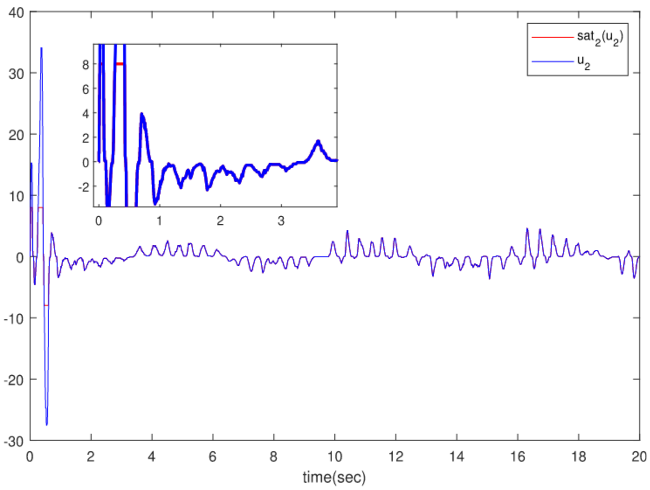

Fig. 9. Input $u_2(t)$ and saturated input $\mathrm{sat}_2(u_2(t))$.

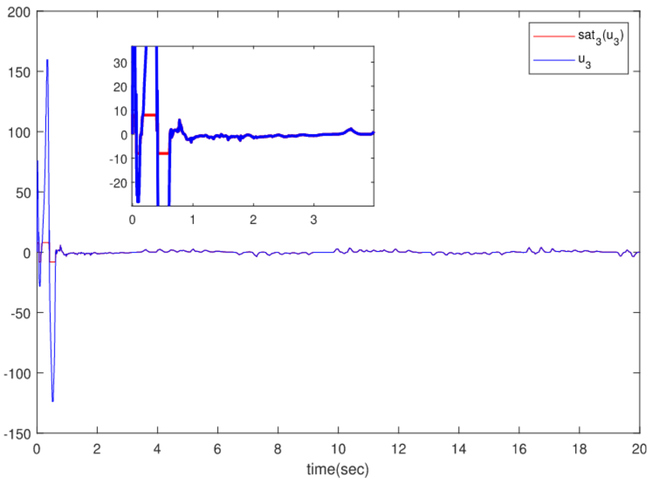

Fig. 10. Input $u_3(t)$ and saturated input $\mathrm{sat}_3(u_3(t))$.

The simulation results are provided in Fig. 2–12. Fig. 2 shows the trajectories of the states $\xi_{i1}, i = 1, 2, 3, 4$, and the results show that it does not violate the restricted bounds. Fig. 3 shows the trajectories of the reference signal ζ_0 and the outputs $\zeta_i, i = 1, 2, 3, 4$. The results show that the event-triggered control method can make $\zeta_i, i = 1, 2, 3, 4$, track the reference signal ζ_0 within a certain range. Fig. 4 shows the trajectories of the reference signal ζ_0 and the outputs $\hat{\zeta}_i, i = 1, 2, 3, 4$. The results show that the event-triggered control method can make $\hat{\zeta}_i, i = 1, 2, 3, 4$, track the reference signal ζ_0 within a certain range. Fig. 5 shows the trajectories of observation errors $\tilde{\xi}_{i1}, i = 1, 2, 3, 4$. The results show that the trajectories of $\tilde{\xi}_{i1}, i = 1, 2, 3, 4$ fluctuate very little in a certain range. Fig. 6 shows the trajectories of the error variable $\eta_{i1}, i = 1, 2, 3, 4$ and the results show

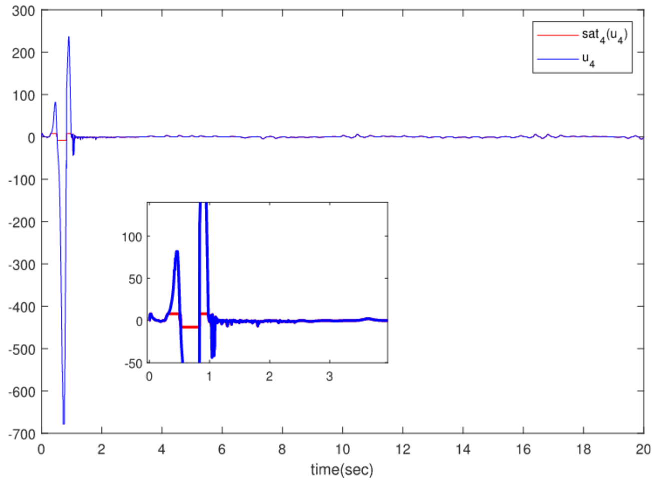

Fig. 11. Input $u_4(t)$ and saturated input $\mathrm{sat}_4(u_4(t))$.

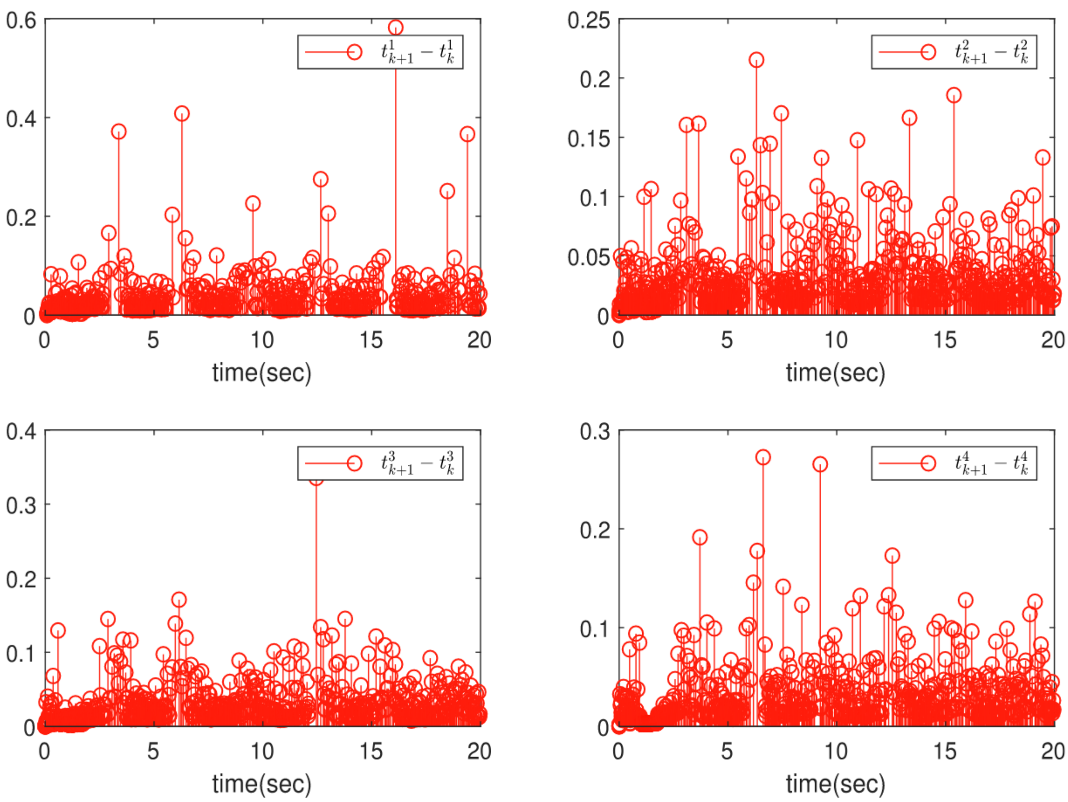

Fig. 12. The trigger time instants t^i_k and inter-event time $t^i_{k+1} - t^i_k$ for agent i.

that it does not violate the restricted bounds. Fig. 7 shows the trajectories of the states ξ_{i2} and the observed states $\hat{\xi}_{i2}, i = 1, 2, 3, 4$. Fig. 8–11, show the trajectories of the control inputs $u_i(t)$ and the saturated input $\text{sat}_i(u_i(t)), i = 1, 2, 3, 4$. Fig. 12 shows the trigger time instants and the inter-event times of four followers respectively.

7. Conclusion

In this paper, an adaptive distributed ETC strategy with an observer has been presented. RBFNN is used to handle the nonlinearity of the system. By constructing a state observer, the unmeasurable states are estimated. A new saturation controller is proposed for SMASs, which is more suitable for practical applications. Besides, the time-varying BLFs are introduced, which can ensure that the partial states constraint conditions are not violated. Considering the benefit of communication resource saving, an adaptive ETC strategy has been proposed to guarantee consensus tracking of SMASs. The proposed ETC strategy can guarantee the boundedness of all system signals, each agent being able to track the given leader signal within a bounded error and avoiding the Zeno behavior successfully. Finally, the correctness of the theoretical results is illustrated by computer simulation.

CRediT authorship contribution statement

Yong Zhao: Methodology, Software, Investigation, Writing - original draft. **Hui Yu:** Conceptualization, Supervision, Visualization, Resources. **Xiaohua Xia:** Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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